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Bounds for the blowup time of the solution to a parabolic system with nonlocal factors in nonlinearities

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ABSTRACT

In this paper, we establish the lower and upper bounds for the blow-up time of the solution to a system of parabolic equations

$$\begin{cases} u_t = \Delta u + u^r(x,t)\displaystyle\int_\Omega v^s(x,t)\mathrm{d}x & \text{in} \quad \Omega\times(0,t^*), \\ v_t = \Delta v + v^q(x,t)\displaystyle\int_\Omega u^p(x,t)\mathrm{d}x & \text{in} \quad \Omega\times(0,t^*) \end{cases}$$

subject to $u(x,t)=v(x,t)=0$ on $\partial\Omega\times(0,t^*)$ and nonnegative initial data under certain assumptions. Here $\Omega\subset\mathbb{R}^n$ is a smooth bounded open domain and $n\geq 3$.

1. Introduction

In this paper, we are interested in the bounds for the blowup time of the solution to the following problem:

$$\begin{cases} u_t = \Delta u + u^r(x,t)\displaystyle\int_\Omega v^s(x,t)\mathrm{d}x, & \text{in } \Omega\times(0,t^*), \\ v_t = \Delta v + v^q(x,t)\displaystyle\int_\Omega u^p(x,t)\mathrm{d}x, & \text{in } \Omega\times(0,t^*), \\ u(x,t)=v(x,t)=0, & \text{on } \partial\Omega\times(0,t^*), \\ u(x,0)=u_0(x), \qquad v(x,0)=v_0(x), & \text{in } \Omega. \end{cases} \tag{1.1}$$

Here $\Omega\subset R^n$ is a smooth bounded open domain and $n\geq 3$, $p,q,r,s\geq 0$. u and v can respectively represent the densities of two biological populations during a migration in the model of population dynamics, or the thickness of two kinds of chemical reactants in the model of chemical reactions, or the temperatures of the two different materials in the model of heat transfer (see [1]).

There are many results about the topic on the bounds for blowup time in nonlinear parabolic problem, we can also refer to [2–16] and the references therein. Very recently, in [17], Payne and Song established the lower bounds for blow-up time

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in the following model of chemotaxis:

$$\begin{cases} u_t = \Delta u - a_1(uv_{,i})_{,i}, & \text{in } \Omega \times (0, t^*), \\ v_t = a_2\Delta v - a_3 v + a_4 u, & \text{in } \Omega \times (0, t^*), \\ \dfrac{\partial u}{\partial \eta} = \dfrac{\partial v}{\partial \eta} = 0, & \text{on } \partial\Omega \times (0, t^*), \\ u(x,0) = u_0(x), \qquad v(x,0) = v_0(x), & \text{in } \Omega, \end{cases} \tag{1.2}$$

where $\Omega \subset \mathbb{R}^3$ (or $\Omega \subset \mathbb{R}^2$) is a smooth bounded domain. Here a comma is used to denote differentiation and the convention of summing over repeated subscripts from 1 to 3 is adopted. In [18], Souplet proved that the solution to (1.1) will blow up for large initial data if $r > 1$ or $q > 1$. Recently, in [19], Liu and Li established that the solution to (1.1) will blow up for large initial data in the case of (i) $r > 1$ or (ii) $q > 1$ or (iii) $ps > (1-r)(1-q)$. And they also discussed the phenomenon of non-simultaneous blowup and obtained the blowup rates. It is a natural way that we are concerned with the bounds for the blowup time of the solution to (1.1). In fact, we will establish both the lower bound and the upper bound for the blowup time of the solution to (1.1). Our results can be applied to estimate the bounds for the solution to (1.1) either in the case of simultaneous blowup or in the case of non-simultaneous blowup.

This paper is organized as follows: In Section 2, we will establish a lower bound for blowup time of the solution to (1.1). And the upper bound for the blowup time will be obtained in Section 3 by different methods in different cases.

2. Lower bound for the blowup time

In this section, we will establish the lower bound for the blowup time t^*.

We define

$$\phi_1(t) = \int_\Omega u^{nl_1}\mathrm{d}x + \int_\Omega v^{nl_2}\mathrm{d}x \tag{2.1}$$

with

$$l_1 > \max\left\{\frac{2(n-2)(r-1)}{n}, \frac{p}{n}, 1\right\}, \qquad l_2 > \max\left\{\frac{2(n-2)(q-1)}{n}, \frac{s}{n}, 1\right\}.$$

Using Green formula, we have

$$\begin{aligned} \phi_1'(t) &= nl_1\int_\Omega u^{nl_1-1}u_t\mathrm{d}x + nl_2\int_\Omega v^{nl_2-1}v_t\mathrm{d}x \\ &= nl_1\int_\Omega u^{nl_1-1}\left(\Delta u + u^r\int_\Omega v^s\mathrm{d}x\right)\mathrm{d}x + nl_2\int_\Omega v^{nl_2-1}\left(\Delta v + v^q\int_\Omega u^p\mathrm{d}x\right)\mathrm{d}x \\ &= nl_1\left(\int_\Omega u^{nl_1-1}\Delta u\mathrm{d}x + \int_\Omega u^{nl_1+r-1}\mathrm{d}x\int_\Omega v^s\mathrm{d}x\right) + nl_2\left(\int_\Omega v^{nl_2-1}\Delta v\mathrm{d}x + \int_\Omega v^{nl_2+q-1}\mathrm{d}x\int_\Omega u^p\mathrm{d}x\right) \\ &= nl_1\left[-(nl_1-1)\int_\Omega u^{nl_1-2}|\nabla u|^2\mathrm{d}x + \int_\Omega u^{nl_1+r-1}\mathrm{d}x\int_\Omega v^s\mathrm{d}x\right] \\ &\quad + nl_2\left[-(nl_2-1)\int_\Omega v^{nl_2-2}|\nabla v|^2\mathrm{d}x + \int_\Omega v^{nl_2+q-1}\mathrm{d}x\int_\Omega u^p\mathrm{d}x\right]. \end{aligned}$$

That is,

$$\begin{aligned} \phi_1'(t) = &-\frac{4(nl_1-1)}{nl_1}\int_\Omega\left|\nabla u^{\frac{nl_1}{2}}\right|^2\mathrm{d}x + nl_1\int_\Omega u^{nl_1+r-1}\mathrm{d}x\int_\Omega v^s\mathrm{d}x \\ &- \frac{4(nl_2-1)}{nl_2}\int_\Omega\left|\nabla v^{\frac{nl_2}{2}}\right|^2\mathrm{d}x + nl_2\int_\Omega v^{nl_2+q-1}\mathrm{d}x\int_\Omega u^p\mathrm{d}x. \end{aligned} \tag{2.2}$$

Applying Young inequality to the second term in the right of (2.2), we get

$$\int_\Omega u^{nl_1+r-1}\mathrm{d}x\int_\Omega v^s\mathrm{d}x \le \frac{1}{\alpha}\left(\int_\Omega u^{nl_1+r-1}\mathrm{d}x\right)^\alpha + \frac{1}{\beta}\left(\int_\Omega v^s\mathrm{d}x\right)^\beta, \tag{2.3}$$

where $1 < \alpha < \frac{4(n-2)}{n}$ and $\frac{1}{\alpha} + \frac{1}{\beta} = 1$.

In convenience, denoting

$$m_1 = \frac{3nl_1(n-2) + n(nl_1) - 4(n-2)(nl_1+r-1)}{3nl_1(n-2) + n(nl_1)} = \frac{n^2l_1 - (n-2)(nl_1+4r-4)}{nl_1(4n-6)},$$

$$m_2 = \frac{4(n-2)(nl_1+r-1)}{3nl_1(n-2)+n(nl_1)} = \frac{2(n-2)(nl_1+r-1)}{nl_1(2n-3)}$$