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A fourth order compact scheme for heat transfer problem in porous media

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ABSTRACT

In this paper, we have extended the recently proposed fourth order compact scheme by Pandit et al. (2007) which was used earlier only for the convection–diffusion equations without considering reaction and nonhomogeneous source terms, and designed to compute flow in a two sided lid-driven differentially heated square cavity filled with a fluid saturated porous medium. The governing Navier–Stokes (N–S) equations in streamfunction–vorticity ($\psi - \zeta$) form of Brinkman-extended Darcy model including the energy transport equation are all solved as a coupled system of equations for the five field variables consisting of streamfunction, vorticity, two velocities and temperature. Moreover, local entropy generation distributions are determined based on the obtained dimensionless velocity and temperature values. The derivative source term present in vorticity equation has been treated as fourth order compact using Padé scheme. The details for the derivation of difference relations at boundaries to generate accurate and stable solutions are also given. We have computed the results for three different cases depending on the direction of moving walls. To assess the numerical accuracy of our proposed scheme, one pertinent test problem with known exact solution is used. The higher order compact scheme adopted in the present study yields consistent performance for variation of key parameters e.g. Richardson number (Ri), Darcy number (Da), Grashof number (Gr) and fixed Prandtl number (Pr)=0.7. The present results are compared with numerical results available in the literature and excellent match is observed in all the cases, establishing the efficiency and accuracy of the extended combined formulations of our fourth order compact scheme and Padé schemes.

1. Introduction

For numerical simulation of fluid dynamical problems, a variety of finite difference methods have been developed over the last few decades. These methods are classified into two groups namely higher order schemes (order of accuracy > 2) and lower order schemes in accordance with desired accuracy of the simulations. Higher order schemes can be constructed based on two basic approaches. One of these two is to use wide stencil by paying the price of tackling some fictitious boundary

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points where no data is available to compute solution. The other one is to use compact stencil [1,2] having no fictitious boundary points. On the other hand, lower order schemes (mostly used second order scheme) do not require fictitious points near the boundary. These schemes require more grid points to resolve the structure of the solution and to attain desired accuracy of the computation. In these situations higher order schemes are still desirable as these schemes require fewer grid points to attain the same accuracy as second order scheme.

Compact schemes characterize high accuracy solution on smaller stencils, so cost effective with favorable numerical stability. In addition to these, Dirichlet and Neumann boundary conditions can be used with less effort. In the spirit of accuracy and compactness (therefore, consuming less memory space), the high order compact schemes have seen increasing popularity in solving computational and geophysical fluid dynamics problems.

There has been a great deal of efforts in recent years to study the fluid flow and heat transfer in rectangular cavities driven by buoyancy and shear. Free convection due to buoyancy forces and forced convection due to shear constitute mixed convection heat transfer, a complex phenomenon due to essential coupling between the fluid flow and heat transfer [3–6]. Moreover, the study of convective flow and heat transfer in fluid saturated porous media [7–14] has been an important topic because of its application in grain storage, chemical catalytic reactors, solar collectors, heat exchangers, solidification of casting, separation processes in chemical industries, etc.

Experimental studies reveal that Darcy flow, non-Darcy flow, turbulent flow, inertial flow, high velocity flow, etc., play a significant role to describe fluid flow phenomena in presence of porous media. For very high velocities in porous media, inertial effects become significant and in such situations, non-Darcy behavior is important for describing fluid flow. Several widely used models [15,16] have been introduced in the literature to study the flow problems in porous media, such as Darcy, the Brinkman-extended Darcy, and the Forchheimer-extended Darcy models. A recent achievement in modeling flow in porous media is the so-called generalized model or Brinkman–Forchheimer-extended Darcy model, in which all fluid forces and the solid drag force are considered in the momentum equation. If one relaxes the hypothesis of negligible inertial effects one gets a series of nonlinear models, depending on the way interaction forces are modeled. It may be mentioned here that Amiri and Vafai in [17] found that excluding the inertial effects would shorten the time needed to reach steady-state. In addition, the results in [18] have shown that inclusion of each of the three effects, i.e. inertia, boundary and velocity-square, reduces the heat transfer rate. These discussions motivate to adopt Brinkman-extended Darcy model by excluding inertia term in this study.

Khanafer and Chamkha [19] have studied numerically on mixed-convection flow in a lid-driven cavity filled with a fluid saturated porous medium. They used Brinkman-extended Darcy model [21,20,22]. Recently, Vishnuvardhanarao and Das [23] have investigated mixed-convection flow in a square cavity filled with porous medium, in such a case the side walls are moving with same velocity in same direction. They used finite volume method using SIMPLE algorithm in a collocated grid arrangement. Also, whenever there has been attempts to solve for the unsteady flows, they are confined invariably to uniform space grids. However, a very little work has been done for the case of unsteady flow situations. Overall, no higher order compact schemes have been found to solve the unsteady mixed-convection with porous medium using nonuniform grids.

The primary motivation is the need for more accurate, unconditionally stable finite-difference solvers for the porous medium equations. In the present work, we have extended the formulation of our recently proposed higher order compact scheme [2] designed for incompressible viscous fluid flows to convective flow and heat transfer on nonuniform grids. However, the equations in this form are essentially the Navier–Stokes equations with a reaction and nonhomogeneous source terms. The source term containing the temperature gradient in the vorticity equation is also discretized using fourth order compact scheme and incorporated into the solution procedure. We have applied our scheme to the problem on mixed-convection flow in a square cavity filled with fluid saturated porous medium, in which both vertical walls are moving, and both horizontal walls are adiabatic. We have considered three different cases depending on the direction of moving walls.

The present study also provides numerically entropy generation due to mixed-convection flow in a square cavity filled with fluid saturated porous medium. In the heat transfer process some amount of useful energy is always lost as a consequence it decreases efficiency of the system. On the basis of second law of thermodynamics this loss of energy due to irreversibility is termed as entropy generation. In this approach, we discretize equations of entropy generation using not only the nodal values of the unknown transport variable but also the values of their first derivatives where these derivatives are discretized by Padé approximation. It is pointed here that in comparison to Nusselt number, entropy generation analysis is relatively a new way to assess the performance of a thermal system and based on the principle of entropy generation minimization [24] to conclude optimum design. There are several studies related to this. In forced convection heat transfer temperature gradient and viscosity effect in the fluid are two main factors contributing to entropy generation. Bejan [25] showed that the temperature gradient and viscosity effect in the fluid are the two main factors contributing to entropy generation for forced convective heat transfer. To the best of authors knowledge, entropy generation analysis during mixed convective flow in a two sided lid driven enclosure filled with fluid saturated porous medium has not been studied so far.

The remainder of this paper is organized as follows. Section 2 gives formulation of the problem. Section 3 describes the discretization of the governing equations with related issues. Section 4 analyzes the solution procedure of algebraic systems. Section 5 deals with the results and discussions and Section 6, the conclusions.