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Study of a problem of functionally graded hollow disk under different thermoelasticity theories—An analysis of phase-lag effects

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ABSTRACT

The present paper is aimed at the investigation of thermo-mechanical interactions inside a functionally graded hollow disk under different thermoelasticity theories in a unified way. The material of the disk is assumed to be graded along its radial direction such that the profiles of the material properties are assumed to follow a volume-fraction based rule with a power law. The inner and outer surfaces of the disk are subjected to different thermal and mechanical boundary conditions. Galerkin type finite element method is used to solve the coupled equations arising out in the present problem in Laplace transform domain. Solution in space–time domain is obtained by employing a numerical inversion method. A detailed analysis highlighting the effects of phase-lags at different times upon the different fields like displacement, temperature, radial stress and hoop stress for different values of the non-homogeneity index is presented by various graphical plots of our numerical results.

1. Introduction

In the field of heat conduction in deformable bodies, several non-classical theories have been proposed in recent years in order to advocate a theory where the propagation of heat is modeled with a finite speed and thereby removing the so called paradox of infinite speed of propagation for thermoelastic disturbances in the classical "coupled dynamical theory of thermoelasticity", derived by Biot [1]. It should be mentioned here that in addition to the paradox of infinite heat propagation speed, the classical dynamic thermoelasticity theory also suffers from the drawback of either unsatisfactory or poor description of a solid's response at low temperature and to a fast transient effects like short laser pulses. These drawbacks mainly arise due to the fact that this theory is based on Fourier law of heat conduction. The non-classical theories of thermoelasticity are therefore developed with the introduction of modified form of Fourier law. Lord and Shulman [2] proposed an extended thermoelasticity theory (ETE) by incorporating a thermal relaxation time in the Fourier law. The temperature-rate dependent thermoelasticity theory (TRDTE) is later on developed by Green and Lindsay [3] with the introduction of two thermal relaxation parameters in the theory although the Fourier law is kept unchanged in this theory. A completely alternative theoretical development in this subject is made by Green and Naghdi [4–6] which have been the center of active research during the last few decades. In this development Green and Naghdi provided sufficient basic modifications in the constitutive equations and proposed three different models of thermoelasticity, labeled as thermoelasticity of types I, II, and III, that cover a much wider class of heat conduction problems.

Recently, Ozisik and Tzou [7] and Tzou [8] proposed a dual phase-lag (DPL) heat conduction law with the introduction of two delay times. One delay time is caused by the micro-structural interactions (small scale effects of heat transport in space,

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such as phonon–electron interaction or phonon scattering) and is termed as phase-lag of temperature gradient. Another delay time τ_q is caused due to the fast-transient effects of thermal inertia (or small scale effect of heat transport in time) and called as phase-lag of heat flux. Both the phase lags are small, positive and assumed to be the intrinsic properties of the medium [8]. The recent models of thermoelasticity are thermoelasticity with dual phase-lags (DPLTE) and the thermoelasticity with three phase-lags (TPLTE). The former model is introduced by Chandrasekharaiah [9] by considering the DPL heat conduction law and the other model is given by Roychoudhuri [10] as a generalized version of type III thermoelasticity [5].

Functionally graded materials (FGMs) are composite media that have continuously changing material properties. FGMs are a new branch of materials developed with the purpose of design of structures to withstand suddenly applied loads. The properties of FGMs vary by a gradual change in composition of the constituent materials through the geometry of the structure. This gradual change in material properties of FGMs offers the use of them in abrupt high exposed loads. The smooth variation of properties within FGMs results in lower stress concentration, intensity factors, higher fracture toughness and improved residual stress distribution as compared with traditional laminated composites. Therefore, FGMs have been widely used as thermal shields, wear-resistant linings, heat exchanger tubes, heat engine components and even prostheses (see Birman and Byrd [11] and Bagri and Eslami [12]). In the last decade a great interest is paid to analyze the response of FGMs under different applied mechanical and thermal loads. Tanigawa [13] analyzed the closed-form solutions of problems with the steady-state condition for the heat conduction problems and thermal stresses that occur in a structure made of non-homogeneous materials. Obata and Noda studied steady thermal stresses in a hollow circular cylinder and a hollow sphere of FGMs using the perturbation method [14] and explained the minimization of the thermal stresses. Lutz and Zimmerman reported the analytical solution for the stresses in FG spheres and cylinders [15,16]. Reddy and Chin reported a thermo-mechanical analysis of FG cylinders and plates [17]. Jeon et al. [18] developed an analytical method to tackle thermoelastic problems for a medium with non-homogeneous properties for shear modulus of elasticity, the thermal conductivity and the coefficient of linear thermal expansion. Vel and Betra [19] obtained an exact solution for three-dimensional deformations of a FG rectangular plate subjected to mechanical and thermal loads. Qian and Betra [20] studied transient thermoelastic deformations of a thick FG plate. Ye et al. [21] investigated the axisymmetric thermoelastic problem of a FG transversely isotropic cylindrical shell. El-Naggar et al. [22] studied the radial deformation and the corresponding stresses in a non-homogeneous orthotropic hollow elastic cylinder. An analytical solution for the FG thick spheres under combined steady mechanical and thermal loads is presented by Eslami et al. [23]. Bakhshi et al. [24] studied the response of FG hollow disk based on the classical theory of thermoelasticity under thermal shock loads. Bahtui and Eslami [25,26] investigated the coupled thermoelasticity and generalized coupled thermoelasticity of FG cylindrical shells subjected to a thermal shock load, respectively. Later on, the response of a FG disk is studied by Bagri and Eslami [27] under the ETE theory. Recently, Carrera et al. [28] investigated the effect of thickness stretching in plate/shell structures made by FGM materials. Golmakani and Kadkhodayan [29] studied large deflection analysis of shear deformable FG plates subjected to thermo-mechanical loads. Sun and Luo [30] presented wave propagation and transient response of an infinite FG circular plate under a point impact load. Ezzat and Atef [31] analyzed a problem of generalized magneto–thermoelasticity interactions in a FG viscoelastic layer (Kelvin–Voigt type) due to the presence of thermal shock in the context of the linear theory of generalized thermoelasticity without energy dissipation. For some recent studies upon FGMs in thermoelasticity theories, the Refs. [32–36] and a recent book by Hetnarski and Eslami [37] may also be mentioned.

This article analyzed the thermoelastic interactions inside a FG hollow disk in the context of the recent thermoelasticity theories. The material of the disk is assumed to be graded through its radial direction such that the profiles of all material properties, except the phase-lags, are assumed to follow a volume-fraction based rule with different non-homogeneity indices. The inner surface of the disk is assumed to be traction free with a thermal exposure while the outer surface is fixed and insulated. The governing equations to tackle are obtained in a coupled form. To solve them numerically, Laplace transform technique with Galerkin finite element method is used. To invert the solution in space–time domain a numerical inversion method proposed by Bellman et al. [38] is employed. The effects of non-homogeneity index, phase-lags at different time upon distributions of different physical fields are investigated by various graphical results.

2. Statement of the problem

We consider a functionally graded hollow disk. The functional material properties of FGMs are generally defined by a relation that correlates the effective material properties of FGM to the constituent material properties. Considering a ceramic–metal FGM in which the composition of the metal and ceramic is introduced by a simple law of mixture of material constituents, the effective material properties of FGM may be defined as

$$P = V_m\,(P_m - P_c) + P_c, \tag{1}$$

where V_m is the volume fraction of the metal, P is the effective property of FGM and the subscripts m and c indicate the metal and ceramic features, respectively. Therefore, P_m and P_c are the properties of metal and ceramic, respectively.

3. Basic governing equations

The governing equations in usual indicial notation for isotropic FGMs in the absence of heat sources and body forces are considered as follows: