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A time domain solution for complex multilayered soil model with circular inhomogeneity by the SBFEM

Shan Lu [a], Jun Liu [a,b,c,*], Gao Lin [a]

[a] School of Hydraulic Engineering, Faculty of Infrastructure Engineering, Dalian University of Technology, Dalian 116024, China
[b] State Key Laboratory of Structural Analysis for Industrial Equipment, Dalian University of Technology, Dalian 116024, China
[c] State Key Laboratory of Ocean Engineering, Shanghai Jiao Tong University, Shanghai, 200240, China

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ABSTRACT

The scaled boundary finite method (SBFEM) is developed for analyzing wave propagation problem in the two-dimensional unbounded domain with rigid bedrock. It combines the advantages of the finite element method and boundary element boundary method. Moreover, the original scaling center is replaced by a scaling line which is more suitable for analyzing the multilayered soil model. Therefore, the modified SBFEM develops the original SBFEM. A new derivation of the modified SBFEM equation is built in the frame of Hamilton system. A continued fraction solution of the dynamic stiffness of the soil model with bedrock is obtained for the first time. Then, by introducing the continued fraction solution and auxiliary variables, it leads to the model resting on bedrock can be solved in time domain. The global equation of motion is solved by the efficient precise time-integration method. This integral method is firstly employed in the modified SBFEM. The precision of the proposed method can achieve computer precision. Therefore, an extremely efficient and accurate solution of the modified SBFEM in time domain is obtained. The results of the complex soil model with circular inhomogeneity show that the proposed method yields excellent results, and high accuracy is observed.

1. Introduction

The analysis of soil–structure interaction problems is of significant importance in elastic dynamics. With few exceptions, numerical solutions are amenable to the homogeneous unbounded soil model. However, there are few appropriate numerical techniques to carry out the solution for the semi-infinite soil model with rigid bedrock in time domain. This attracts more and more attentions of engineers. Over the past decades, different numerical techniques have been used to develop the unbounded soil–structure interaction problem. Such common techniques include the finite element method (FEM) [1,2], boundary element method (BEM) [3–5], the thin layer method (TLM) [6–10], the exact Dirichlet-to-Neumann (DtN) mapping [11] and the infinite element method [12]. It is well accepted that, among them, the finite element method is the most suitable and versatile in handling the unbounded domain problem. However, the challenges in terms of computer running time and human effort in generating finite element models are encountered with some engineering applications. For modeling the radiation damping of the unbounded domain soil model, very fine meshes have to be generated in the

* Corresponding author at: School of Hydraulic Engineering, Faculty of Infrastructure Engineering, Dalian University of Technology, Dalian 116024, China. Tel.: +86 013610921908 (Mobile).
E-mail address: liujun8128@126.com (J. Liu).

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near field in order to obtain accurate results. Especially, the degrees of freedom of the engineering practice model are huge. Therefore, the generation of a high-quality finite element mesh is a time-consuming work.

Recently, various methods have been employed to improve the computational efficiency. The BEM is an attracting method for solving the unbounded soil–structure interaction problem, because only boundary is discretized, which results in a reduction of the spatial dimension by one. It is widely used in many fields, such as nonlinear problem, stress singularity problem and unbounded domain problem. However, there are some difficulties exist in applying this method to some practical engineering problems. Because the BEM depends on the fundamental solution, and the solution is hard to obtain in some special case. Meanwhile, the fundamental solution in frequency domain should be expressed in the forms of the complex mathematical formulations. In particular, the fundamental solution of the complex soil model with rigid bedrock is hard to achieve. The TLM is suitable for analyzing the model with the horizontal layered media. However, it is incorrect when the thickness of layer is too big. So, the TLM is only employed to simulate the model with simple geometry and homogeneous material.

Recently the semi-analytical scheme named the scaled boundary finite element method (SBFEM) [13,14] has employed successful application to the soil–structure interaction problems. Song and Wolf are the founders of this method, and then systematically describe the SBFEM in Ref. [15]. In the following years, the SBFEM has obtained continually and huge development. Combining the advantages of BEM and FEM, only the boundary is discretized with surface finite element in the SBFEM. Therefore, the spatial dimension is reduced by one, at the same time, no fundamental solution and artificial boundary conditions are required. In particular, the anisotropic model can be easily modeled by SBFEM without any difficulty.

The increasing popularity SBFEM has been expanded to analyze the frequency domain problems [16–20]. Then, the SBFEM was intended to solve time domain problems. At the early stage of time domain analysis, the governing equation was solved through integrating the unit impulse response. It is a consuming time process, especially for the large size problem. Soon, some researchers proposed an efficient linear approximation method for solving convolution [21–24]. Recently, Song [25] and Birk et al. [26,27] have developed the continued fraction method for bounded domain problems, acoustic and diffusion problems in frequency domain. Subsequently, the continued fraction method was successfully introduced in time domain problems for bounded and unbounded domains by Chen [28,29] and Birk [30]. However, as above mentioned the time domain solution of the complex inhomogeneous model resting on rigid bedrock is seldom obtained. The aim of this paper is to study the dynamic response of the complex inhomogeneous model with bedrock in time domain.

Since Eshelby's research [31], the inhomogeneity problems have received a number of attentions. By using the equivalent inhomogeneity method, the inhomogeneous problems were solved. Eshelby's method has been extensively applied to investigate many kinds of inhomogeneous problems [32,33]. However, the closed form solutions can be obtained only for the simple geometry model with inhomogeneity. Hence, the solutions of the complex inhomogeneous model can only be obtained by the numerical methods, such as, the volume integral method (VIM) [34–36], the FEM [37–39] and the boundary integral equation method (BIEM) [40] and others methods. However, due to the FEM using domain discretization for obtaining more accurate solutions, the computational efficiency is quite low. The VIM is suitable for analyzing the model with embedded inhomogeneity. It only needs to discretize the part of inhomogeneity. This greatly simplifies the process of calculation. But, the VIM cannot be extended to the anisotropic medium owing to the complex fundamental solutions. Finally, the BEIM is more efficient method in analyzing complex inhomogeneity problems. But, the boundary integral equation has to be formulated for each inhomogeneity. It is inconvenient for modeling the complex model with many inhomogeneities. In this paper, the modified SBFEM is applied to solve the complex soil model with inhomogeneity. The modified SBFEM overcomes the above difficulties for analyzing the inhomogeneity problems.

In this paper, the dynamic stiffness equation of soil layer with rigid bedrock is obtained, which is based on Hamilton system firstly. Then, the solution of dynamic stiffness is achieved by using the continued fraction method. The continued fraction method is firstly expressed for the unbounded domain with rigid bedrock. Through introducing auxiliary variables, the global equation of motion is built. To solve the equation, many step-by-step integration methods are proposed, such as the Newmark method and Wilson-θ method [41,42]. However, there are the exponent overflow and diverge problems for them. To obtain more accurate numerical results, the precise integration method (PIM) is proposed by Zhong [43–45] who is the earliest researcher to pay attention to this problem. Because the integrate time step is divided into small pieces, and five terms Taylor expansion can satisfy the precision for each integral interval. The method can achieve computer precision and avoid round-off error. It enormously improves the accuracy and computational efficiency of the modified SBFEM in time domain.

The further outline of this paper is as follows: In Section 2, the concept and fundamental equation of the modified SBFEM based on the Hamilton system is introduced in detail. In Section 3, the dynamic stiffness equation for the soil layer resting on rigid bedrock is developed. In Section 4, the equation of motion of unbounded domain is constructed firstly by using the continued fraction solution and introducing auxiliary variables. Then, the equation of motion of bounded domain is built. By the interaction force on boundary, the bounded and unbounded domains are coupled. The global equation of motion of the soil model with rigid bedrock is obtained. In Section 5, the precise time integration method is firstly applied to solve the global equation of motion of the modified SBFEM. In Section 6, five examples are solved and compared with other methods in frequency and time domains, so that the accuracy and practicability of the proposed method can be checked. The conclusions are remarked in Section 7.