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# Numerical study of partial slip on the MHD flow of an Oldroyd 8-constant fluid

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**Abstract**

The steady flow of an Oldroyd 8-constant magnetohydrodynamic (MHD) fluid is considered for a cylindrical geometry when the no-slip condition between the cylinders and the fluid is no longer valid. The inclusion of the partial slip at boundaries modifies the governing boundary conditions, changing from a linear to a non-linear situation. The non-linear differential equation along with non-linear boundary conditions governing the flow has been solved numerically using a finite-difference scheme in combination with an iterative technique. The solution for the no-slip condition is a special case of the presented analysis. A critical assessment is made for the cases of partial slip and no-slip conditions.


*Keywords:* Partial slip; Rheological parameters; Oldroyd 8-constant fluid

## 1. Introduction

We recall that fluids in which the shear stress is a multiple of the shear strain are called Newtonian fluids. The proportionality coefficient is the viscosity. Other fluids are known as non-Newtonian fluids. Examples of Newtonian fluids are: water, alcohol, benzene, kerosene and glycerine. Examples of non-Newtonian fluids are: blood plasma, chocolate, tomato sauce, mustard, mayonnaise, toothpaste, asphalt, some greases and sewage.

The governing equation that describes the flow of a Newtonian fluid is the Navier–Stokes equation. During the past several years, generalizations of the Navier–Stokes model to highly non-linear constitutive laws have been proposed and studied because of their interest in applications. There is not a single governing equation which exhibits all the properties of non-Newtonian fluids and these fluids cannot be described simply as Newtonian fluids. Moreover, there are very few cases in which the exact analytic solution of Navier–Stokes equations can be obtained. These are even rare if the constitutive equations for the non-Newtonian fluids are considered. One of the popular models for non-Newtonian fluids is the model that is called the Oldroyd 8-constant fluid. It is reasonable to use the Oldroyd 8-constant fluid model to see the rheological effects even for unidirectional and steady flow. It is pertinent to mention here that unidirectional flows of an Oldroyd 3-constant fluid (Rajagopal and Bhatnagar [1], Hayat et al. [2] and Fetecau and Fetecau [3–5], Tan and Masuoka [6] and Chen et al. [7]) take into account the rheological effects in an

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unsteady situation only and lack the features of viscoelasticity for the steady state situation. Baris et al. [8] considered an Oldroyd 8-constant model to discuss the steady flow in a convergent channel. In continuation, Hayat et al. [9] discussed the Couette, Poiseuille and generalized Couette flows of an Oldroyd 8-constant magnetohydrodynamic fluid.

In all the mentioned studies, the effect of the slip condition is not considered. Navier [10] proposed a slip boundary condition when the slip velocity depends on the shear stress. He developed slip boundary conditions based on molecular calculations. There is much rigorous work [11–20] concerning the flow of a Navier–Stokes-slip, threshold-slip, etc. Since the equations for non-Newtonian fluids are of higher order than the Navier–Stokes equations, additional boundary conditions are necessary in order to obtain the unique solution. The adherence boundary conditions are insufficient to determine a unique solution. Rajagopal and Gupta [21] and Rajagopal and Kaloni [22] gave examples of non-uniqueness in domains with porous boundaries. This implies that additional boundary conditions are necessary to ensure the well-posedness, but it remains an open question what boundary conditions should be imposed. Moreover, non-Newtonian fluids such as polymer melts often exhibit wall slip. The fluids exhibiting boundary slip have important technological applications. For example, the polishing of artificial heart valves and internal cavities in a variety of manufactured parts is achieved by imbedding such fluids with abrasives [23]. Several attempts have been made to explain slip phenomena [24–27]. Examples of well-posedness results for the Navier–Stokes equations with Navier slip, and more references, are given in [28–31]. Rao and Rajagopal [32] also examined the effect of the slip boundary condition on the flow of fluids in a channel. Roux [33] studied in detail the existence and uniqueness of the flow of second grade fluids with slip boundary conditions. Non-Newtonian flows with wall slip have been studied numerically in Refs. [34–42]. The effect of the slip condition at the wall for Couette flow for steady and unsteady state conditions has been studied respectively by Jha [43] and Marques et al. [44], and for Stokes and Couette flows by Khaled and Vafai [45].

The object of the present analysis is to examine the partial slip effects on an MHD Oldroyd 8-constant fluid between coaxial cylinders. The conducting fluid is permeated by an imposed uniform magnetic field when the no-slip condition at the boundaries is invalid. The inclusion of the partial slip at boundaries modifies the governing boundary conditions, changing from a linear to a non-linear situation. The highly non-linear problem has been solved numerically, and the results have been discussed in detail. The considered Hartman flow of an electrically conducting fluid in the presence of a transverse magnetic field has applications in many devices such as MHD power generators, MHD pumps, and accelerators; in processes such as aerodynamics heating, electrostatic precipitation, polymer technology; and in the purification of molten metals from nonmetallic inclusions and fluid-droplet sprays.

## 2. Governing equations

Consider the flow of an incompressible magnetohydrodynamic (MHD) fluid. The magnetic field is applied transversely to the flow. The following set of pertinent field equations governing the unsteady motion of the conducting Oldroyd 8-constant fluid is given by

$$\rho\left[\frac{\partial \mathbf{V}}{\partial t}+(\mathbf{V}\cdot\nabla)\,\mathbf{V}\right]=\operatorname{div}\mathbf{T}+\mathbf{J}\times\mathbf{B}, \tag{1}$$

$$\operatorname{div}\mathbf{V}=0, \tag{2}$$

$$\nabla\cdot\mathbf{B}=0,\quad \nabla\times\mathbf{B}=\mu_m\mathbf{J},\quad \nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}, \tag{3}$$

$$\mathbf{J}=\sigma\left(\mathbf{E}+\mathbf{V}\times\mathbf{B}\right), \tag{4}$$

$$\mathbf{T}=-p\mathbf{I}+\mathbf{S}, \tag{5}$$

$$\begin{aligned}&\mathbf{S}+\lambda_1\frac{D\mathbf{S}}{Dt}+\frac{\lambda_3}{2}\left(\mathbf{S}\mathbf{A}_1+\mathbf{A}_1\mathbf{S}\right)+\frac{\lambda_5}{2}\left(\operatorname{tr}\mathbf{S}\right)\mathbf{A}_1+\frac{\lambda_6}{2}\left[\operatorname{tr}\left(\mathbf{S}\mathbf{A}_1\right)\right]\mathbf{I}\\&\quad=\mu\left[\mathbf{A}_1+\lambda_2\frac{D\mathbf{A}_1}{Dt}+\lambda_4\mathbf{A}_1^2+\frac{\lambda_7}{2}\left[\operatorname{tr}\left(\mathbf{A}_1^2\right)\right]\mathbf{I}\right],\end{aligned} \tag{6}$$

$$\mathbf{A}_1=\mathbf{L}+\mathbf{L}^{\mathsf{T}},\quad \mathbf{L}=\operatorname{grad}\mathbf{V}. \tag{7}$$