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Statistical analysis of bubble and crystal size distributions: Formulations and procedures

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Abstract

Bubble and crystal size distributions have previously been described only by either exponential or power law functions. Within this limited framework, it has not been possible to characterize size distributions in a fully quantitative manner. We have developed an analytical and computational formulation with which to characterize and study crystal and bubble size distributions (BSD). This formulation demonstrates that all distributions known to date belong to the logarithmic family of statistical distributions. Four functions within the logarithmic family are best suited to natural bubbles and crystals (log normal, logistic, Weibull, and exponential). This characterization is supported by the fact that the power law function widely used for crystal and bubble size analysis is not a statistical distribution function, but rather represents an approximation of the upper regions (larger bubbles/crystals) of the logistic distribution, whose sizes are much larger than the mode.

The coefficients for each of the four logarithmic functions can be derived by 1) best fit exceedance function of the logarithmic distribution, and 2) best fit of the linear transformation of the distribution probability density. A close match of the coefficients derived by the above two methods can be used as an indicator of correct function fitting (choice of initial values). Function fitting by exceedance curves leads to the most accurate statistical results, but has certain strict limitations, including 1) a requirement to rescale the base distribution function; 2) a higher failure rate for function fitting than that for distribution density; 3) uncertainty in observational data error estimates; and 4) unsuitability for visual interpretation. The most productive approach to visualization and interpretation of size distributions is through linear transformation of logarithmic distributions on the basis of probability densities. This also makes it possible to 1) clearly discern bimodal distributions; 2) assess the range of observed objects relative to the full range of the indicated distribution; 3) determine number densities for each mode directly; and 4) integrate to obtain total volume fraction for comparison with available observations. The latter could, in some cases, provide more accurate results than many measurement methods.

Unambiguous definition of Bubble Number Density (BND) must be based on the number of bubbles per melt volume (not number of bubbles per bulk volume), so that like is done with crystals, it can be directly used as an indicator of basic

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vesiculation processes such that: a) nucleation leads to increase of BND, b) diffusive or decompressive bubble growth keeps BND constant, and c) coalescence decreases BND.

Keywords: bubble size distribution; crystal size distribution; statistical analysis; bubble number density; crystal number density; exceedance; magma chamber processes; eruption dynamics; vesicular lavas

1. Introduction

Bubble and crystal size distributions have been studied for many years (Sahagian, 1985; Marsh, 1988; Toramaru, 1989; Bisperink and Ronteltap, 1992), as they reveal processes that are not directly observable, such as flow in magma chambers and conduits, nucleation events, and the history of parcels of magma throughout the eruption process and in lava flows at the surface (Sahagian et al., 1989; Cashman et al., 1992; Proussevitch et al., 1993; Cashman et al., 1994; Vergniolle, 1996; Marsh, 1998). Previous studies of bubble and crystal size distributions have found that many can be characterized by exponential (Marsh, 1988) or power law (Gaonac'h et al., 1996a) functions. However, these are special cases of more general distributions. The data upon which these studies were based, as well as our own new data from vesicular lavas (Proussevitch et al., 2007-this issue; Sahagian et al., 2002) show that they all fall in the family of logarithmic distributions. In this paper, we develop a more universally applicable formulation and present a methodology for treating all such size distributions.

1.1. Background

Our present generalized analysis is built upon the shoulders of seminal studies that have been conducted in the past. Pioneering work that explored the physics of crystal nucleation and growth dynamics to derive an analytical formulation for crystal size distribution was conducted by Marsh in the late 1980's (Marsh, 1988). This work predicted an exponential distribution for single episode crystal nucleation combined with crystal growth. Coefficients for the distribution functions were directly linked to growth rates. This "classic" formulation has been subsequently applied to numerous studies of bubble size distributions (Sarda and Graham, 1990; Cashman and Mangan, 1994; Cashman et al., 1994; Blower et al., 2003).

In another study, Toramaru developed an analytical formulation for bubble nucleation and growth rates and applied it to a numerical model to predict bubble size distributions (Toramaru, 1989). He imposed eight different initial conditions (e.g. depth, decompression rate, initial dissolved water concentration, etc.) in the model to determine bubble size distribution. However, this mechanistic approach did not lead to any statistical interpretation of distribution functions (although it is evident upon inspection that they are logarithmic distributions). The theoretical results were subsequently applied to a number of vesicular lavas ranging in composition from basalt to rhyolite with the goal of reconstructing the physical conditions and processes within the magma body that led to the observed distributions (Toramaru, 1990).

In a later study, Cashman applied the formulation of Marsh (Marsh, 1988) in an attempt to characterize the bubble size distributions of Kilauean basalts (Cashman and Mangan, 1994). It was not possible, however, to characterize the full distribution because only large bubbles were available for analysis. It was not possible to include the smaller part of the distribution because the methodology of counting bubbles from photographs of their cross-sections could not resolve the small bubbles. As a result, the individual bubbles were larger than the mode of the actual distribution.

A very thorough analysis of power law function was subsequently conducted by Gaonac'h (Gaonac'h et al., 1996a,b; Lovejoy et al., 2004; Gaonac'h et al., 2005). This function can be effectively used to characterize the upper part of the bubble size spectrum (when smaller bubbles are neglected). While this is adequate for it application to a limited part of the distribution, the power law formulation is actually a special case of a log logistic distribution used by statisticians for other applications. We explore log logistic distributions and their application to bubble size distributions in Section 7, below.

More recently, Blower et al. explored evolution of exponential and power law functions to describe the bubble distributions of observed samples (Blower et al., 2001, 2003). They formulated a model for single and multiple nucleation events and subsequent bubble growth and found that the distribution resulting from multiple nucleation events that are common for silicic systems could be characterized by variations in the power function. This is in contrast to the interpretations of Gaonac'h who attributed the distribution to coalescence (Gaonac'h et al., 1996b). In basaltic melts where vesiculation