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Comparisons of several algorithms for Toeplitz matrix recovery

Chuan-Long Wang [a], Chao Li [a,*], Jin Wang [b]

[a] Higher Education Key Laboratory of Engineering Science Computing in Shanxi Province, Taiyuan Normal University, Taiyuan 030012, PR China

[b] School of Computer Science and Technology, Beihang University, Beijing 100191, PR China

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ABSTRACT

In this paper, we study algorithms for Toeplitz matrix recovery. Inspired by the singular value thresholding (SVT) algorithm for matrix completion and the alternating directions iterative method, we first propose a new mean value algorithm for Toeplitz matrix recovery. Then we apply our idea to the augmented Lagrange multiplier (ALM) algorithm for matrix recovery and put forward four modified ALM algorithms for Toeplitz matrix recovery. Convergence analysis of the new algorithms is discussed. All the iterative matrices generated by the five algorithms keep a Toeplitz structure that ensures the fast singular value decomposition (SVD) of Toeplitz matrices. Compared with the original algorithms, our algorithms are far superior in the time of SVD, as well as the CPU time.

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1. Introduction

The study of recovering a corrupted low-rank matrix has experienced amazing growth in recent years. This problem is well known as the matrix recovery (MR) problem, as well as the Robust PCA. It arises in a large number of application areas [1–3].

MR problem was first proposed by Wright [4] and Candès [5]. In [4] Wright showed that a low-rank matrix A from $D = A + E$ with sufficiently errors E can be exactly recovered under rather broad conditions by solving the following convex optimization problem,

$$\begin{aligned} \min_{A,E} \quad & \|A\|_* + \lambda \|E\|_1 \\ \text{s.t.} \quad & D = A + E \end{aligned} \tag{1.1}$$

where $\|A\|_* = \sum_{k=1}^{r} \sigma_k(A)$, $\sigma_k(A)$ denotes the kth largest singular value of $A \in \mathbb{R}^{n_1 \times n_2}$ of rank r. $\|E\|_1$ denotes the sum of the absolute values of matrix entries, and λ is a positive weighting parameter. In [4,5], the best choice of λ is $\frac{1}{\sqrt{n_1}}$. Throughout this paper, unless otherwise specified, we will fix $\lambda = \frac{1}{\sqrt{n_1}}$.

Many algorithms have been proposed to solve the optimization problem (1.1). Wright et al. [4] presented an iterative thresholding (IT) algorithm, which requires a large number of iterations to converge. Then Lin et al. [6,7] gave two new algorithms for solving the optimal problem (1.1), one is the accelerated proximal gradient (APG) algorithm; the other is the

* Corresponding author.
E-mail addresses: clwang1964@163.com (C.-L. Wang), l_chao1989@163.com (C. Li), wjyz91@163.com (J. Wang).

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dual algorithm. Both the APG algorithm and the dual algorithm are at least 50 times faster than the IT algorithm. In 2010, Lin et al. [8] put forward the augmented Lagrange multiplier (ALM) algorithm, which has been proved to have a Q-linear convergence speed.

On the other hand, as an important special matrix, Toeplitz matrices arise naturally in certain application areas such as the system identification [9], medical imaging [10], the multiple-input multiple-output (MIMO) communication system [11], image restoration [12]. Therefore, some scholars have studied Toeplitz matrices, such as Shaw et al. [13], Kailath et al. [14]. It is worth mentioning that Qiao et al. [15,16] put forward an $O(n^2 \log n)$ algorithm for the fast SVD of Toeplitz and Hankel matrices by combining the Lanczos method [17] and the FFT technique [18].

We can see from the foregoing algorithms for the matrix recovery problem that most of the algorithms need to compute SVD, which is time-consuming and accounts for at least 85% of the CPU time. Therefore, we can take full advantage of the fast SVD of Toeplitz matrices to reduce computational complexity, as well as the CPU time. Together with the value of Toeplitz matrices in the signal and image processing, it is very meaningful to study Toeplitz matrix recovery problem.

In this paper, we focus our attention on the recovery of Toeplitz matrices. Combining the idea of the mean value algorithm for Toeplitz matrix completion [19] and the alternating directions iterative method, we first propose a new mean value algorithm for Toeplitz matrix recovery. Then we present four modified ALM algorithms for Toeplitz matrix recovery. First, we give some definitions.

Definition 1 (*[17]*)**.** An $n \times n$ Toeplitz matrix $T \in \mathbb{R}^{n\times n}$ is of the form,

$$T = \begin{pmatrix} t_0 & t_1 & \cdots & t_{n-2} & t_{n-1} \\ t_{-1} & t_0 & \cdots & t_{n-3} & t_{n-2} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ t_{-n+2} & t_{-n+3} & \cdots & t_0 & t_1 \\ t_{-n+1} & t_{-n+2} & \cdots & t_{-1} & t_0 \end{pmatrix}.$$

Note: T is determined by its first row and first column, a total of $(2n-1)$ entries.

Definition 2 (*Singular Value Decomposition (SVD) [17]*)**.** The singular value decomposition of a matrix $X \in \mathbb{R}^{n_1\times n_2}$ of rank r is:

$$X = U\Sigma_r V^*, \quad \Sigma_r = \mathrm{diag}(\sigma_1, \ldots, \sigma_r),$$

where $U \in \mathbb{R}^{n_1\times r}$ and $V \in \mathbb{R}^{n_2\times r}$ are orthogonal, $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_r > 0$.

Definition 3 (*Singular Value Thresholding Operator [20]*)**.** For each $\tau \geq 0$, the singular value thresholding operator $\mathcal{D}_\tau$ is defined as follows:

$$\mathcal{D}_\tau(X) := U\mathcal{D}_\tau(\Sigma)V^*, \quad \mathcal{D}_\tau(\Sigma) = \mathrm{diag}(\{\sigma_i - \tau\}_+)$$

where $X = U\Sigma_r V^*$ is the SVD of a matrix X of rank r, $\{\sigma_i - \tau\}_+ = \begin{cases} \sigma_i - \tau, & \text{if } \sigma_i > \tau \\ 0, & \text{if } \sigma_i \leq \tau. \end{cases}$

Definition 4 (*Soft-Thresholding (Shrinkage) Operator [8]*)**.** For each $\varepsilon \geq 0$, the soft-thresholding (shrinkage) operator $\mathcal{S}_\varepsilon$ is defined as follows:

$$\mathcal{S}_\varepsilon[x] = \begin{cases} x - \varepsilon, & \text{if } x > \varepsilon, \\ x + \varepsilon, & \text{if } x < -\varepsilon, \\ 0, & \text{otherwise,} \end{cases}$$

where $x \in \mathbf{R}$.

The rest of this paper is organized as follows. In Section 2, we describe our algorithms for Toeplitz matrix recovery in detail and their convergence is established in Section 3. In Section 4, we compare our algorithms with the ALM algorithm and SVT algorithm through numerical experiments. Conclusions are given in Section 5.

Notation. For convenience, $\mathbf{R}$ denotes the set of real numbers. $\mathbb{R}^{n_1\times n_2}$ denotes $n_1 \times n_2$ real matrices set. $r(X)$ denotes the rank of a matrix X. x_{ij} denotes the (i,j)th entry of a matrix X. The nuclear norm of a matrix is denoted by $\|X\|_*$, the Frobenius norm by $\|X\|_F$, $\|X\|_1$ denotes the sum of the absolute values of matrix entries, and $|X|_0$ denotes the number of nonzero elements of a matrix X. X^* is the conjugate transpose of a matrix X. The standard inner product of two matrices is denoted by $\langle X, Y\rangle = \mathrm{trace}(X^*Y)$. $\Omega = \{-n_1+1, \ldots, n_2-1\}$ are the indices of diagonals of a matrix X. Vector $\mathrm{diag}(X, l)$ denotes the lth diagonal of a Toeplitz matrix X, $l \in \Omega$. The mean value of a vector x is denoted by $\mathrm{mean}(x)$, the median value by $\mathrm{median}(x)$.

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