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Radial basis function partition of unity methods for pricing vanilla basket options

Victor Shcherbakov*, Elisabeth Larsson

Uppsala University, Department of Information Technology, Box 337, SE-751 05 Uppsala, Sweden

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ABSTRACT

Meshfree methods based on radial basis function (RBF) approximation are becoming widely used for solving PDE problems. They are flexible with respect to the problem geometry and highly accurate. A disadvantage of these methods is that the linear system to be solved becomes dense for globally supported RBFs. A remedy is to introduce localisation techniques such as partition of unity. RBF partition of unity methods (RBF–PUM) allow for a significant sparsification of the linear system and lower the computational effort. In this work we apply a global RBF method as well as RBF–PUM to problems in option pricing. We consider one- and two-dimensional vanilla options. In order to price American options we employ a penalty approach. A penalty term, suitable for American multi-asset call options, has been designed. RBF–PUM is shown to be competitive compared with a finite difference method and a global RBF method. It is as accurate as the global RBF method, but significantly faster. The results for RBF–PUM look promising for extension to higher-dimensional problems.

1. Introduction

Option contracts have been used for many centuries, but trading of options, as well as academic research on option pricing, increased dramatically in volume after 1973, when Black and Scholes published their market model [1]. Nowadays a variety of options are traded at the world exchanges, starting with simple vanilla options and continuing to multi-dimensional index options. Therefore, there is a high demand for correct option prices. Moreover, option prices play an important role in risk management, hedging, and parameter estimation.

In this paper we consider the problem of pricing so called vanilla basket options, i.e., European and American options, with several underlying assets. A European option is a contract with a fixed exercise date, while an American option can be exercised at any time before maturity. Among the different available models of the underlying behaviour, such as the Heston model with stochastic volatility or the Merton model with jump diffusion, we select the standard Black–Scholes model, since it is a basic test case. Under the Black–Scholes model the price of European and American options can be determined by solving either a partial differential equation or a stochastic differential equation [2]. In the case of a single-asset European option the price is known analytically, while for multi-assets options the prices have to be computed numerically. The American option is more difficult due to the opportunity to exercise the option at any time. Such an opportunity introduces a free exercise boundary, which complicates the problem. The price for an American option needs to be computed numerically even in the single-asset case.

* Corresponding author.
E-mail addresses: victor.shcherbakov@it.uu.se (V. Shcherbakov), elisabeth.larsson@it.uu.se (E. Larsson).

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There are several techniques to handle the free exercise boundary. The most commonly used technique consists in rewriting the free boundary problem as a linear complementarity problem (LCP) and then solving it by a standard method, such as projected successive over-relaxation (PSOR) [3]. The drawback of this method is that it is relatively slow. Another method, that is used in industry, is the operator splitting (OS) method [4]. It is fast and effective for one-dimensional problems. Alternatively, a penalty approach can be taken as proposed in [5], and further developed in [6–8]. A penalty term designed to approximately enforce the early exercise condition is added to the PDE, which allows for removing the free boundary and solving the problem on a fixed domain. In combination with radial basis function (RBF) methods, variations of the penalty approach have been popular for handling American options, see [9–13]. It is also possible to ignore the free boundary in each time step, and then apply the American constraint explicitly. This has been done for RBF methods in [14–16]. In this paper, we evaluate the performance of the penalty approach in the RBF setting with respect to accuracy and computational cost.

There are various numerical methods, which are used for option pricing in industry as well as in academia. Perhaps the most popular methods are Monte Carlo (MC) methods [17] and finite difference (FD) methods [3]. Both of them have their own strengths and weaknesses. MC methods converge slowly but are effective for pricing high-dimensional options, because the computational cost scales linearly with the number of underlying assets. On the other hand, FD methods have a better convergence rate, while the computational cost grows exponentially with the number of underlying assets. Other types of methods that are used are binomial tree methods [18] and Fourier expansion based methods [19].

We aim to construct a method for option pricing, based on radial basis function approximation, that can be competitive for low-dimensional to moderately high-dimensional problems. RBF methods can achieve high order algebraic, or for some problems even exponential, convergence rates [12,20]. It means that in order to get the same accuracy the problem size will be smaller than with FD, which is crucial if we work in a many-dimensional space. A global RBF method was shown to compare favourably with an adaptive FD method in [21] in one and two dimensions.

Another advantage of RBF methods is that they are meshfree and therefore can accommodate non-trivial geometries. In financial applications, the computational domains that are used in the literature are often regular. For example, squares, cubes or hypercubes can easily be used. However, depending on the nature of the contract function, using another shape of the domain can lead to substantial computational savings, see, e.g., [21], where a simplex domain is used instead. Furthermore, with a meshfree method, the discretisation can easily be adapted to resolve local features in the solution.

A drawback of global RBF methods is that the linear system that needs to be solved is dense and often ill-conditioned. The situation can be improved by introducing localisation techniques, see e.g., [22–24]. One way to introduce locality is to employ a partition of unity framework, which was proposed by Babuška and Melenk in 1997 [25]. A partition based formulation is also well suited for parallel implementation. Some work on parallelisation for localised RBF methods has been done, see for example [23,26,27]. The ill-conditioning can be addressed by, for example, the RBF-QR technique [28–30].

In this paper we consider the problem of pricing dividend paying vanilla basket call options. In order to solve the problem we use global RBF and RBF partition of unity methods (RBF–PUM). We show that RBF based methods provide a good alternative to already existing methods. All comparisons of the solutions are made against a standard FD solution for European options an FD-OS solution for American options.

The outline of the paper is as follows. In Section 2, we introduce the Black–Scholes model for European and American basket call options. In Section 3, we discuss the penalty approach for American options and its form in the case of call options. Then in Section 4, we give an overview of RBF methods and RBF–PUM. Section 5 contains numerical experiments and comparisons. Finally, Section 6 concludes the paper.

2. The Black–Scholes model

The multi-dimensional Black–Scholes equation takes the form

$$\frac{\partial V}{\partial t} = \mathcal{L}V, \quad \mathbf{x} \in \Omega,\ t \in (0, T], \tag{2.1}$$

where V is the value of the option, $\mathbf{x} = (x_1, \dots, x_d)$ defines the spot prices of the d underlying assets, Ω is the domain of definition, t is the backward time, i.e., time to maturity, and T is the maturity time of the option. The spatial operator $\mathcal{L}$ takes the form

$$\mathcal{L} = \frac{1}{2}\sum_{i,j=1}^{d} \Sigma_{ij} x_i x_j \frac{\partial^2}{\partial x_i \partial x_j} + \sum_{i=1}^{d} (r - D_i) x_i \frac{\partial}{\partial x_i} - r, \tag{2.2}$$

where D_i is the continuous dividend yield paid out by the ith asset, the matrix $\Sigma = [\sigma\sigma^*]$, where σ is the volatility matrix, and r is the risk-free interest rate.

The payoff function for the call option is given by:

$$\Phi(\mathbf{x}) = \max\left(\sum_{i=1}^{d} \alpha_i x_i - K, 0\right), \tag{2.3}$$