Computers and Mathematics with Applications 71 (2016) 201–212

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Three-dimensional nonlinear modified Zakharov–Kuznetsov equation of ion-acoustic waves in a magnetized plasma

Aly R. Seadawy *

Mathematics Department, Faculty of science, Taibah University, Al-Ula, Saudi Arabia
Mathematics Department, Faculty of Science, Beni-Suef University, Egypt

ARTICLE INFO

Article history:
Received 5 July 2015
Received in revised form 10 October 2015
Accepted 9 November 2015
Available online 2 December 2015

Keywords:
Magnetized plasma
Ion-acoustic waves
Extended direct algebraic method
Modified Zakharov–Kuznetsov equation

ABSTRACT

The nonlinear three-dimensional modified Zakharov–Kuznetsov (mZK) equation governs the behavior of weakly nonlinear ion-acoustic waves in a plasma comprising cold ions and hot isothermal electrons in a presence of a uniform magnetic field. By using the reductive perturbation procedure leads to a mZK equation governing the propagation of ion dynamics of nonlinear ion-acoustic waves in a plasma. The mZK equation has solutions that represent solitary traveling waves. We found the electrostatic field potential and electric field in form traveling wave solutions for three-dimensional mZK equation. The solutions for the mZK equation are obtained precisely and efficiency of the method can be demonstrated. The stability of solitary traveling wave solutions of the mZK equation to three-dimensional long-wavelength perturbations are investigated.

1. Introduction

If the plasma is magnetized, the governing equation when the electrons are isothermal is the Zakharov–Kuznetsov (ZK) equation [1]. The ZK equation arises in number of scientific models including fluid mechanics, astrophysics, solid state physics, plasma physics, chemical kinematics, chemical chemistry, optical fiber and geochemistry [2–5]. The stability of plane-periodic and solitary traveling-wave solutions of the ZK equation to two-dimensional long-wavelength perturbations has been investigated [6,7]. Li et al. [8] examined the ZK equation and obtained exact traveling wave equations by using the extended tanh method and direct assumption method. As a result of study in [8], kink-shaped and bell-shaped solitons were formally derived. For more details about the solitary wave solutions and the ZK equation, the reader is advised to read [9–11].

Nonlinear propagating dust-acoustic solitary waves (DASWs) in a warm magnetized dusty plasma containing different size and mass negatively charged dust particles, isothermal electrons, high- and low-temperature ions were investigated. For this purpose, a reasonable normalization of the hydrodynamic and Poisson equations was used to derive the ZK equation for the first-order perturbed potential. As the wave amplitude increases, the width and the velocity of the solitons deviated from the prediction of the ZK equation. To described the soliton of larger amplitude, a linear inhomogeneous ZK-type equation for the second-order perturbed potential was derived. Stationary solutions of both equations were obtained using the renormalization method. Numerically, the effect of power law distribution on the higher-order corrections was examined [12].

The ZK equation is a very attractive model equation for the study of vortices in geophysical flows. The ZK equation appears in many areas of physics, applied mathematics and engineering. In particular, it shows up in the area of Plasma Physics [13,14]. The ZK equation governs the behavior of weakly nonlinear ion-acoustic waves in a plasma comprising of

* Correspondence to: Mathematics Department, Faculty of Science, Beni-Suef University, Egypt.
E-mail address: Aly742001@yahoo.com.

http://dx.doi.org/10.1016/j.camwa.2015.11.006
0898-1221/

cold ions and hot isothermal electrons in the presence of a uniform magnetic field [15,16]. A three-dimensional extended Zakharov–Kuznetsov (EZK) equation was derived for small but finite amplitude nonlinear Langmuir structures, by using the hydrodynamic equations of electrons and positrons and Poisson equation with stationary ions. The EZK equation was solved analytically and the features of the nonlinear excitations are investigated numerically [12]. Solitary waves solutions were generated by deriving the nonlinear higher order of extended KdV equations for the free surface displacement [13]. By using fractional sub-equation method, The analytical exact solutions of some nonlinear evolution equations in mathematical physics; namely the space–time fractional Zakharov–Kuznetsov and modified Zakharov–Kuznetsov equations were obtained [14].

Munro and Parkes [17] showed that if the electrons are non-isothermal, the governing equation of the ZK equation is a modified form, refereed to as the mZK equation. They also showed that with an appropriate modified form of the electron number density proposed by Schamel [18], a reductive procedure leads to a modified form of the ZK equation. The mZK equation has solutions that represent plane-periodic and solitary traveling waves propagating. Munro and Parkes investigated the stability of these solutions to long-wavelength perturbations in an arbitrary direction by using the method described [6].

The mZK equation in the electrical transmission line is investigated [4]. Different expressions on the parameters in the mZK equation were given. By means of the Hirota method, bilinear forms and soliton solutions of the mZK equation were obtained. Linear-stability analysis yields the instability condition for such soliton solutions. Phase-plane analysis was conducted on the mZK equation for the properties at equilibrium points. The perturbed mZK equation, which can be proposed when the external periodic force was considered. Both the weak and developed chaotic motions were observed [4].

The electromagnetic solitary structures, which are found to exist in such a degenerate EP plasma, are significantly modified by the effects of degenerate electron and positron pressures. The applications of the results in an EP plasma medium, which occurs in compact astrophysical objects [19,20]. Traveling wave analysis is given in [21] for the ZK equation. Soliton solutions are derived using the improved modified extended tanh-function method [20]. One-dimensional soliton, apparently inelastic, periodic solutions and N-soliton solutions have been obtained. The auxiliary equation method and the direct Hirota bilinear method were applied to the quantum ZK equation in [22–25].

This paper is organized as follows: in Section 1, an introduction is given. In Section 2, the problem formulations of the propagation in a three-dimensional of weakly nonlinear ion-acoustic waves in a plasma comprising cold ions and hot isothermal electrons in a presence of a uniform magnetic field is formulated. The nonlinear three-dimensional modified Zakharov–Kuznetsov equation is derived. In Section 3, the electric field potential and electric field in form of solitary wave solutions of the mZK equation are obtained and analyzed. Finally the paper end with a conclusion in Section 4.

2. Derivation of a modified ZK equation

The propagation in a three-dimensional of weakly nonlinear ion-acoustic waves in a plasma comprising cold ions and hot isothermal electrons in a presence of a uniform magnetic field is considered. The ion dynamics of nonlinear ion-acoustic waves in a plasma are governed by the following fluid system of equations [17]:

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{u}) = 0, \tag{1}$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla\phi + \mathbf{u} \wedge \Omega \mathbf{e}_x, \tag{2}$$

$$\nabla^2\phi = n_e - n, \tag{3}$$

$$n_e = \exp\phi, \tag{4}$$

where n is the non-dimensional ion number density; n_e is the electron number density; $\mathbf{u}$ is the ion fluid velocity; ϕ is the electrostatic potential; Ω is the uniform external magnetic field, the wave is assumed to propagate in the x direction and the direction is specified by the unit vector $\mathbf{e}_x$. Eq. (1) is the continuity equations, the equations of motion represented in Eq. (2), the system is closed by the Poisson equation (3), the electrons are free and isothermal in Eq. (4). In the non-dimensionalized form, the reference density is the unperturbed number density n_0; the speed is the ion sound speed $c_s = \sqrt{K_B T_{ef}/m_i}$, κ_B is the Boltzmann's constant, T_{Fe} is the constant temperature of the free electrons, m_i is the ion mass; the length $\lambda_D = c_s\omega_{pi}$, $\omega_{pi} = \sqrt{n_0 e^2/\epsilon_0 m_i}$ is ion plasma frequency, $-e$ is the electron charge, ϵ_0 is the vacuum permittivity; the electrostatic potential is $K_B T_{ef}/e$.

In order to find a suitable choice of scaling for the independent variables, the reductive perturbation method to obtain the modified ZK that governs the behavior of small amplitude ion-acoustic waves is used. The following scaling for the independent variables are stretched as

$$\xi = \epsilon^{1/4}(x-t), \qquad \eta = \epsilon^{1/4}y, \qquad \zeta = \epsilon^{1/4}z, \qquad \tau = \epsilon^{3/4}t, \tag{5}$$

where ϵ is the small parameter characterizing the typical amplitude of the waves. To investigate the basic properties of the nonlinear equations for the reduced model (1)–(4), we use the singular perturbation methods applied to weakly nonlinear