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Numerical schemes for pricing Asian options under state-dependent regime-switching jump–diffusion models

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A B S T R A C T

We study the pricing problem of Asian options when the underlying asset price follows a very general state-dependent regime-switching jump–diffusion process via a partial differential equation approach. Under this model, the price of the option can be obtained by solving a highly complex system of coupled two-dimensional parabolic partial integro-differential equations (PIDEs). We prove existence of the solution to this system of PIDEs by the method of upper and lower solutions via constructing a monotonic sequence of approximating solutions whose limit is a strong solution of the PIDE system. We then propose several numerical schemes for solving the system of PIDEs. One of the proposed schemes is built upon the constructive proof, hence its results are *provably* convergent to the solution of the system of PIDEs. We illustrate the accuracy of the proposed methods by several numerical examples.

1. Introduction

Asian options are financial derivatives whose payoffs depend on the average of the underlying asset price over a pre-specified time period. These options are very popular because they can reduce the risk of market manipulation of the underlying instrument at maturity. A major challenge in pricing Asian options is that the distribution of the average asset price is usually not available analytically, even under the simple Geometric Brownian Motion (GBM) model, except in the special case of geometric average. Hence, numerical methods must be used to price these financial derivatives, e.g. Monte-Carlo methods of Broadie et al. [1]; Glasserman [2]; Kemna and Vorsta [3], numerical partial differential equation (PDE) techniques in [4,5], dimension reduction techniques proposed in [6,7], perturbation methods developed in [8,9], and the Laplace inversion method of German and Yor [10].

All the papers discussed above are within the Black–Scholes (BS) framework. Despite its popularity, it is well known that the BS model suffers from several deficiencies, such as inconsistencies with the market-observed implied volatility smile (or skew). Many extensions to the BS model have been introduced in the literature to provide more realistic descriptions for asset price dynamics. In particular, the BS model has been extended to account for empirical behavior of implied volatility smile (smirk). Among the most popular extensions are regime-switching and jump–diffusions. One way to introduce additional randomness into the BS model is by incorporating a finite-state Markov chain into the GBM model, resulting in the so-called Markovian regime-switching models [11]. By allowing the model parameters, such as the volatility, rate of

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return, interest rate, and dividend rate, to take different values in different regimes, regime-switching models can capture more accurately the change in macro-market conditions, while, at the same time, preserve to a certain degree the simplicity of the model. In addition, as illustrated in, for example, Yao et al. [12], regime switching models are able to generate implied volatility smile commonly found in empirical studies. While the literature on pricing European/American options under regime-switching models is quite rich, only a handful of papers in the open literature discuss the pricing of Asian options under regime-switching models, such as Boyle and Draviam [13]; Yuen and Yang [14], and very recently, Chan and Zhu [15].

Another important direction of BS model extensions is to augment the GBM with jumps, as pioneered in [16]. The literature on pricing Asian options under jump models is relatively less developed compared to that under the simple BS model. Papers in the mainstream include [17], where a semi-Lagrangian method is proposed to solve the partial integro-differential equation (PIDE) directly, or Vecer and Xu [18], where a representation for Asian options under semi-martingale models is developed via analysis of the PIDE. Building on Vecer and Xu [18], the technique proposed in [19] involves constructing a sequence of functions that converges uniformly to the option price. Along a different line of research, a double-Laplace transform method for pricing Asian options under the double exponential jump–diffusion model of Kou [20] is developed in [21].

This paper is primarily concerned with pricing of Asian options under models that incorporate *both* regime-switching and jump–diffusions, hereinafter referred to as regime-switching jump–diffusion models, with state-dependent switching rate. We emphasize the following two relevant shortcomings in the existing literature which formed the motivation for this work. Firstly, although there has been considerable interest in applying regime-switching jump–diffusion models to various financial problems, the existing literature on pricing options under these models solely focuses European/American options. Examples include modeling electricity prices [22], short rates [23], portfolio selection [24], and option pricing [25–31,24]. Secondly, the common assumption in the literature on Markovian regime-switching models that the Markov chain is independent of the Brownian motion is not realistic, especially for options written on major indexes (e.g. S&P500). More specifically, under this assumption, the switching of regimes can change the behavior of the asset price, but the asset price does not have any influence on how fast or slow the regime switches. Although this independence assumption appears to be acceptable for options written on individual stocks, since we may assume that the impact of the price change of the particular stock on the overall market is negligible, it is not realistic for options written on major indexes. This is because, in this case, the index itself can be an indicator of market conditions, and thus, the influence of the index on the overall market needs to be taken into account. For this reason, when pricing options written on major indexes under a regime-switching model, it is highly desirable to relax the above-mentioned independence assumption by allowing the regime-switching to depend on the underlying asset process. We refer the reader to the recent work by Yin and Zhu [32] for a detailed discussion of this new and important class of stochastic models. To the best of our knowledge, pricing Asian options under regime-switching models with state-dependent switching rate has *not* been previously studied in the literature.

In this paper, we present PDE numerical schemes for pricing Asian options when the underlying process follows a very general state-dependent regime-switching jump–diffusion model. Under this model, the price of the option can be obtained by solving a highly complex system of *coupled* two-dimensional parabolic PIDEs, and hence iterative techniques must be employed to solve it. To the best of our knowledge, there does not exist a *provably* convergent PDE-based algorithm for solving such a system of PIDEs in the context of pricing Asian options. In this paper, we prove the convergence of one numerical scheme to the true solution of the PDE system. We believe that the results presented in this paper are interesting, new, and can be used as a comparison in future studies. In short, our paper makes the following major contributions:

1. We study, for the first time, via a PDE approach, the pricing problem of Asian options under a very general state-dependent regime-switching jump–diffusion model. In this model, the regime generator is allowed to be state-dependent, and different regimes can have different jump-distributions.
2. We prove existence of the solution to the highly complex system of coupled two-dimensional parabolic PIDEs that arises. Our proof technique is built upon the method of upper and lower solutions via constructing a monotonic sequence of approximating solutions whose limit is a strong solution of the PIDE system.

   We propose several schemes to solve this system of coupled PIDEs. One of the proposed schemes is suggested by the constructive proof, and hence its numerical solutions are *guaranteed* to converge to the solution of the system of PIDEs. In addition, this scheme allows for a very simple and natural parallelization of the solution process on multi-core architectures.
3. We provide numerical results for Asian options under various highly complex regime-switching jump–diffusion models which have never been studied in the literature. Hence, these results can be used as references for future research in pricing Asian options.

   In particular, our numerical results suggest that the state-dependency of the switching rate can have considerable effects on the option hedging parameters, and could make hedging under a state-dependent model very different from a state-independent one.

As an advantage, the algorithms described in this paper may be implemented using any PDE solver. In our work, we used PDE2D, a popular general-purpose PDE solver written by the third author Sewell [33]. The PDE2D solver uses up to fourth degree finite elements to solve very general systems of nonlinear, steady-state, time-dependent and eigenvalue PDEs in 1D intervals, general 2D regions, and a wide range of simple 3D regions, with general boundary conditions. It has a graphical