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A novel dynamic multilevel technique for image registration

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ABSTRACT

We present a novel dynamic multilevel technique for solving image registration problems. The development is carried out to construct a spatial transformation based on cubic B-spline basis functions and determine the control points dynamically. Unlike FEM-based image registration methods, we do not have the difficulty of solving a complicated matrix system. In addition, the presented method is enhanced by a multilevel technique, which makes it more efficient and flexible. The numerical results and several comparison studies on real bio-medical images show our technique is stable, accurate and fast, especially for large deformation registration problems.

1. Introduction

Image registration techniques, developed in recent years, aim to align two images by finding a spatial transformation [1–3]. These methods fall mainly into three basic categories, the landmark-based registration [4–6], the segmentation-based registration [7–9] and the image intensity-based registration [10,11]. They have many applications such as medical imaging [12,13], remoting sensing [14,15] and computer vision [16]. In addition to various types of spatial transformations, image registration approaches can also be divided into rigid registration and non-rigid (or deformable) registration. The landmark-based registration first identifies a limit set of landmark points on the target image, and also some corresponding salient points on the reference image. This step can be done manually or automatically. Then the spatial transformation can be defined based on the relationship among these points. The next step is to minimize the spatial transformation, which is also known as the orthogonal Procrustes problem [17,18]. The segmentation-based registration methods utilize segmented parts of the reference image and the target image and seeks to align them together. Apart from the landmark-based and the segmentation-based methods, the intensity-based registration methods directly operate on the image intensity values. The spatial transformations are based on the intensity information recorded in both images, combining the selected basis functions and transferring the discrete digital image data into a continuous expression. From a theoretical point of view, the intensity-based registration methods are the most flexible because they use all the available information throughout the registration process.

Our current study is based on the work of several pioneers for intensity-based registration [19,20]. In [19], a complicated image registration model is proposed, and two spatial transformations are defined for each of the images. Then an energy function is defined by three components: one fidelity term and two regularization terms. To obtain the parameters in the spatial transformations, the energy function is minimized and a finite element system is developed. The proposed

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registration method in [19] has shown promising performances, demonstrating more accurate results than a traditional linear interpolation method and an optical flow-based method. But the computational requirements for the proposed method in [19] are more intensive than the others. This is because the matrix assembly and solving the resulting large system are expensive. Therefore in our study, we still keep the same energy function. Instead of using the traditional finite element method (FEM) [21–26], we directly work on the energy function, simplifying the complicated finite element matrix system.

Our proposed registration method aligns two images automatically based solely on the intensity information. The presented image registration model is a parametric model, looking for a moderate number of parameters. In our study, these parameters are called control points, and they are calculated by a dynamic strategy. The spatial transformation is the combination of the cubic B-spline basis functions with the to-be-solved control points as the coefficients. We also enhance the registration model with a novel multilevel technique. The coarser levels mainly deal with the initial large deformation. To control the error, we switch to finer levels in the matrix system. These finer levels will capture more detailed information and improve the registration accuracy. For the quantitative study, we define a similarity ratio to measure the effects of our algorithm. Some artificial images and real medical images are used to test our method in the final numerical study. The numerical experiments show that the presented technique is flexible, stable, accurate and efficient in dealing with image registration problems. Unlike FEM-based methods [19], our contributions in this paper include

1. We present a relatively simple approach to deal with image registration problems. It does not involve a large matrix system assembly and solving, because we are working on the energy functional directly. Therefore, the method performs much faster;
2. A novel multilevel technique is first combined with the dynamic model, making the model more flexible; and
3. The presented technique handles large deformable registration very well, and it is also more stable and efficient. The large deformations are interpreted as large differences between the target image and the reference image in our current study.

This paper is organized as follows. In Section 2 we present our dynamic mathematical modeling technique for image registration. Then, the multilevel technique and the full algorithm are presented in Section 3. We apply our technique to several examples in Section 4. This paper ends with a brief summary in Section 5.

2. Dynamic mathematical modeling

Suppose two images, the reference image $I_1(\mathbf{x})$ and the target image $I_0(\mathbf{x})$ are given, image registration aims to find a continuous spatial transformation $f(\mathbf{x})$ such that $I_1(f(\mathbf{x})) \approx I_0(\mathbf{x})$. The spatial transformation is written as

$$f : \mathbb{R}^2 \to \mathbb{R}^2$$
$$f(\mathbf{x}) = \sum_{i=1}^{N} \mathbf{C}_i \phi_i(\mathbf{x}), \tag{1}$$

where $\mathbf{C}_i$ are the control points, $\phi_i(\mathbf{x})$ are the corresponding global basis functions, and N is the number of global basis functions applied to the registration model. The global basis functions are a combination of the cubic B-splines in two dimensions. Cubic B-splines are the most common basis functions used in image processing studies. They have many good properties, such as small overlap, local support and C^2 smoothness. These advantages of cubic B-splines have been studied and compared with harmonic functions, radial basis functions and wavelets [11]. The recursive formula of B-splines can be written as,

$$N_{i,0}(u) = \begin{cases} 1 & \text{if } u \in [u_i, u_{i+1}) \\ 0 & \text{otherwise}, \end{cases} \tag{2}$$

and

$$N_{i,p}(u) = \frac{u - u_i}{u_{i+p} - u_i} N_{i,p-1}(u) + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}(u), \tag{3}$$

where $N_{i,p}$ represents the ith B-spline function of polynomial order $p + 1$. It is defined on a knot vector $\{u_1, \ldots, u_m\}$ in the u direction. Likewise in 2D, there is a sequence of B-splines in the v direction associated with the knot vector $\{v_1, \ldots, v_n\}$. When $p = 0$, $N_{i,0}$ becomes a step function and equals zero everywhere except on the half open interval $[u_i, u_{i+1})$.

In this paper, we focus on 2D image registration problems. The basis functions ϕ_i in Eq. (1) are a combination of cubic B-splines, and the cubic B-splines in each direction are defined on open knot vectors, which means the starting and ending knots are repeated by $p + 1$ times. Thus $N_{j,3}(u)$ are the cubic B-splines in the u direction with an open knot vector $\xi^u = \{u_1, u_1, u_1, u_1, u_2, \ldots, u_{m-1}, u_m, u_m, u_m, u_m\}$, and $N_{k,3}(v)$ are the cubic B-splines in the v direction with an open knot vector $\xi^v = \{v_1, v_1, v_1, v_1, v_2, \ldots, v_{n-1}, v_n, v_n, v_n, v_n\}$. Then the global basis functions $\phi_i(\mathbf{x})$ are expressed by

$$\phi_i(\mathbf{x}) = N_{j,3}(u) N_{k,3}(v), \tag{4}$$