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14-velocity and 18-velocity multiple-relaxation-time lattice Boltzmann models for three-dimensional incompressible flows

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ABSTRACT

In this paper, 14-velocity and 18-velocity multiple-relaxation-time (MRT) lattice Boltzmann (LB) models are proposed for three-dimensional incompressible flows. These two models are constructed based on the incompressible LBGK model proposed by He et al. (2004) and the MRT LB model proposed by d'Humières et al. (2002), which have advantages in the computational efficiency and stability, respectively. Through the Chapman–Enskog analysis, the models can recover to three-dimensional incompressible Navier–Stokes equations in the low Mach number limit. To verify the present models, the steady Poiseuille flow, unsteady pulsatile flow and lid-driven cavity flow in three dimensions are simulated. The simulation results agree well with the analytical solutions or the existing numerical results. Moreover, it is found that the present models show higher accuracy than d'Humières et al. model and better stability than He et al. model.

1. Introduction

The lattice Boltzmann method is an innovative approach based on kinetic theory to simulate various complex fluid systems [1,2]. The significant advantages of lattice Boltzmann method are the natural parallelism of algorithm, simplicity of programming and ease of dealing with complex boundary conditions. It has been successfully applied in the field of complex fluids, such as multiphase fluids [3], microfluidics [4], fluids in porous media [5,6], and impinging fluids [7,8].

Until now, the lattice Bhatnagar–Gross–Krook (LBGK) model, which is based on a single-relaxation-time (SRT) approximation [9], is still the most popular LB model due to its extreme simplicity. The earliest LBGK model is proposed by Qian et al. [10], which is often used to simulate the incompressible flow in the low Mach number limit. However, through the Chapman–Enskog (C–E) procedure, this model can only recover to the compressible Navier–Stokes (N–S) equations in the low Mach number limit. If the density fluctuation is assumed to be negligible, the incompressible N–S equations can be derived. But in practical simulations, sometimes the density fluctuation cannot be neglected. In this case, there is compressible effect in the simulations and this effect may lead to serious numerical errors. In fact, Qian et al. model can be viewed as a compressible scheme to simulate incompressible flows. There are efforts to weaken the low Mach number restriction of Qian et al. model to extend this model for compressible flows [11,12], while in our paper we are focused on how to reduce the compressible effect in existing LB models.

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In order to reduce the compressible effect in Qian et al. model, many other LB models have been proposed [13–16]. Among them, the models proposed by He and Luo, and Guo et al. are widely used. The basic idea of He–Luo model [14] is to neglect the terms of higher order Mach number in equilibrium distribution function, which can explicitly reduce the compressible effect as demonstrated in the following simulations in Ref. [14]. However, He–Luo model can only accurately recover to the incompressible momentum equations, but keeps the term $\frac{1}{c_s^2}\partial p/\partial t$ in the continuity equation, where $p = c_s^2\rho/\rho_0$ is the normalized pressure. When He–Luo model is applied to the unsteady flow, it requires an additional condition $T \gg L/c_s$ (T and L are characteristic time and length, respectively), to eliminate the compressible effect.

As we know, the incompressible limit is equivalent to low Mach number limit. To overcome the shortcoming of He–Luo model, Guo et al. proposed a new LBGK model [15] for two-dimensional incompressible flows. Guo et al. model can exactly recover to the incompressible N–S equations only in the low Mach number limit, which is accomplished by completely decoupling the pressure and density and delicately designing the equilibrium distribution function. To our knowledge, Guo et al. model is the first LBGK model which can be applied to steady and unsteady incompressible flows while simultaneously eliminating the compressible effect completely. Due to the advantage of Guo et al. model, He et al. extended this model to three dimensions and proposed the three-dimensional 15-velocity and 19-velocity LBGK models for incompressible flows [16].

Although the LBGK model has a very simple algorithm and is popularly used, its stability is not always satisfactory in the practical applications. To overcome this shortcoming, many other LB models have been developed in the past few years [17–24]. Among them, the MRT LB model [22–24] has received the most attention due to its superior numerical stability. d'Humières firstly proposed the MRT LB model [22] at nearly the same time with Qian et al. model. Lallemand and Luo carried out detailed analysis on this type of model [23] and found that it has much better performance than the LBGK model in the stability. To further demonstrate the superior stability of MRT model over the LBGK model, d'Humières et al. developed three-dimensional 15-velocity and 19-velocity MRT models [24].

The MRT model has much better stability than the LBGK model, but in the aspect of computational efficiency the MRT model could be about 15% slower than the LBGK counterpart in terms of the number of nodes updated per second [24]. The update of one node includes the memory access and the floating-point operations, so the computational efficiency of MRT LB schemes is mostly limited by the memory access quantity and the calculation amount. To improve the computational efficiency of the present MRT models, we propose a new class of three-dimensional MRT models with less memory consumption and calculation amount in this paper.

The MRT model proposed by d'Humières et al. for three-dimensional incompressible flows [24] is based on the Qian et al. model or He–Luo model, which both use q discrete velocity directions in d dimensions, i.e., use the DdQq lattice models. However, it is noticed that the incompressible LBGK model proposed by He et al. [16] only uses $q-1$ discrete velocity directions in the actual computation. Moreover, d'Humières et al. MRT model contains a moment corresponding to kinetic energy square, which does not affect the recovered macroscopic N–S equations. Therefore, based on the He et al. model and d'Humières et al. model, it is possible to construct an MRT model with a $(q-1)\times(q-1)$ transformation matrix, which can reduce the memory consumption and enhance computation efficiency of the existing MRT models in three dimensions.

The above idea is enlightened by the work of Du and Shi, who proposed a two-dimensional 8-velocity incompressible (iD2Q8) MRT model [25] based on Guo et al. LBGK model and the two-dimensional MRT model proposed by Lallemand and Luo. As a continuing work, we propose two three-dimensional MRT models with 14-velocity and 18-velocity based on He et al. LBGK model and d'Humières et al. MRT model in this paper. The general construction method of $(q-1)\times(q-1)$ transformation matrix is presented. Through the C–E procedure, the proposed models can recover to the incompressible N–S equations in the low Mach number limit. The numerical results of unsteady pulsatile flow and cavity flow show that the proposed model is more accurate than d'Humières et al. MRT model and more stable than He et al. LBGK model, where d'Humières et al. MRT model and He et al. LBGK model are two widely used LB models for three-dimensional incompressible flows. As an example, only the 14-velocity model is presented in details in this paper.

The rest of the paper is organized as follows. Section 2 briefly describes the three-dimensional 15-velocity incompressible (iD3Q15) LBGK model proposed by He et al. Section 3 presents our three-dimensional 14-velocity incompressible (iD3Q14) MRT model. We provides the simulation results for three benchmark problems: the three-dimensional Poiseuille flow, pulsatile flow and cavity flow by using the proposed iD3Q14 MRT model in Section 4. We also compared some results with those coming from d'Humières et al. D3Q15 MRT model and He et al. iD3Q15 LBGK model. Section 5 concludes this paper. Appendix A briefly give the derivation of incompressible N–S equations from iD3Q14 MRT model. Appendix B outlines the three-dimensional 18-velocity incompressible (iD3Q18) MRT model.

2. iD3Q15 LBGK model proposed by He et al.

The iD3Q15 LBGK model proposed by He et al. in Ref. [16] includes 15 discrete velocity directions as follows:

$$\{\mathbf{c_0}, \mathbf{c_1}, \ldots, \mathbf{c_{14}}\} = \begin{pmatrix} 0 & 1 & -1 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \end{pmatrix} c$$

where $c = \delta_x/\delta_t$ is the particle velocity and δ_x and δ_t are the lattice spacing and time step, respectively.