Computers and Mathematics with Applications 63 (2012) 1020–1025

Contents lists available at SciVerse ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

RH-conservative matrix characterization of P-convergence in probability

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ARTICLE INFO

Article history:
Received 1 August 2010
Received in revised form 20 October 2011
Accepted 20 October 2011

Keywords:
RH-conservative
Pringsheim limit point
P-convergent
Random variables
P-convergence in probability

ABSTRACT

The goal of this paper is to characterize P-convergence in probability of four-dimensional weighted means using RH-conservative matrices. We begin with the presentation of the following theorem. Let $(X_{k,l}) = (X_k X_l)$ be a double sequence of non-degenerate independently identically distributed random variables such that $\mathrm{E}(X_{k,l}) = \mu$ and $\mathrm{E}(X_{k,l}) < \infty$ for each (k, l). Suppose that $A = (a_{m,n,k,l})$ is an RH-conservative matrix; then the necessary and sufficient condition for $Y_{m,n}$ to P-converge to $\mu(a - \sum_{k,l} c_{k,l}) + \sum_{k,l} c_{k,l} X_{k,l}$ in probability is that

$$\text{P-}\lim_{m,n} \sup_{k,l} |a_{m,n,k,l} - c_{k,l}| = 0.$$

Other variations and implications will also be presented.

1. Introduction

In 1986 Das and Mohanty presented the following generalization of Jamison, Orey, and Pruitt's result. Let (X_k) be a sequence of non-degenerate independently identically distributed random variables such that $E(X_k) = \mu$ and $E(X_k) < \infty$ for each k. Suppose that $A = (a_{n,k})$ is a conservative matrix; then the necessary and sufficient condition for Y_m to converge to $\mu(a - \sum_k c_k) + \sum_k c_k X_k$ in probability is that

$$\lim_n \sup_k |a_{n,k} - c_k| = 0.$$

The main goal of this paper is to present a multidimensional analog of Das and Mohanty's results. We begin with the following theorem. Let $(X_{k,l}) = \{X_k X_l\}$ be a double sequence of non-degenerate independently identically distributed random variables such that $E(X_{k,l}) = \mu$ and $E(X_{k,l}) < \infty$ for each (k, l). Suppose that $A = (a_{m,n,k,l})$ is an RH-conservative matrix; then the necessary and sufficient condition for $Y_{m,n}$ to P-converge to $\mu(a - \sum_{k,l} c_{k,l}) + \sum_{k,l} c_{k,l} X_{k,l}$ in probability is that

$$\mathrm{P} - \lim_{m,n} \sup_{k,l} |a_{m,n,k,l} - c_{k,l}| = 0.$$

Throughout this paper we use a multidimensional analog of Das and Mohanty's methods to establish the theorem here. We have also presented, in addition to the above theorem, variations and implications of this theorem.

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doi:10.1016/j.camwa.2011.10.057

2. Definitions, notation and preliminary results

Definition 2.1 (*[1]*)**.** A double sequence $x = [X_{k,l}]$ has the *Pringsheim limit* L (denoted by $\mathrm{P} - \lim x = L$) provided that given $\epsilon > 0$, there exists $N \in \mathbf{N}$ such that $\left|X_{k,l} - L\right| < \epsilon$ whenever $k, l > N$. Such an x is described more briefly as "P-convergent".

Definition 2.2 (*[2]*)**.** The double sequence y is a *double subsequence* of x provided that there exist increasing index sequences $\{n_j\}$ and $\{k_j\}$ such that if $x_j = x_{n_j,k_j}$, then y is formed by

$$\begin{array}{cccc} x_1 & x_2 & x_5 & x_{10} \\ x_4 & x_3 & x_6 & - \\ x_9 & x_8 & x_7 & - \\ - & - & - & -. \end{array}$$

In [3], Robison presented the following notion of a conservative four-dimensional matrix transformation and a Silverman–Toeplitz type characterization of such a notion.

Definition 2.3. The four-dimensional matrix A is said to be *RH-conservative* if it maps every bounded P-convergent sequence into a P-convergent sequence.

Theorem 2.1 (*[4,3]*)**.** *The four-dimensional matrix A is RH-conservative if and only if:*

RH-C$_1$: $\mathrm{P} - \lim_{m,n} a_{m,n,k,l} = c_{k.l}$ *for each k and l;*
RH-C$_2$: $\mathrm{P} - \lim_{m,n} \sum_{k,l=1,1}^{\infty,\infty} a_{m,n,k,l} = a$*;*
RH-C$_3$: $\mathrm{P} - \lim_{m,n} \sum_{k=1}^{\infty} \left|a_{m,n,k,l} - c_{k,l}\right| = 0$ *for each l;*
RH-C$_4$: $\mathrm{P} - \lim_{m,n} \sum_{l=1}^{\infty} \left|a_{m,n,k,l} - c_{k,l}\right| = 0$ *for each k;*
RH-C$_5$: $\sum_{k,l=1,1}^{\infty,\infty} \left|a_{m,n,k,l}\right| < A$ *for all* (m, n)*; and*
RH-C$_6$*: there exist finite positive integers A and B such that*
$\sum_{k,l>B} \left|a_{m,n,k,l}\right| < A$*.*

When these conditions $\mathrm{RH} - \mathrm{C}_1$ – $\mathrm{RH} - \mathrm{C}_6$ are satisfied, we have

$$\mathrm{P} - \lim_{m,n} Y_{m,n} = \mu \left(a - \sum_{k,l} c_{k,l}\right) + \sum_{k,l} c_{k,l} X_{k,l}$$

where $\mu = \mathrm{P} - \lim_{k,l} X_{k,l}$ and the double series $\sum_{k,l=1,1}^{\infty,\infty} c_{k,l}(X_{k,l} - \mu)$ is always P-convergent. Note that if $c_{k,l} = 0$ for all (k, l) and $a = 1$ then Theorem 2.2 reduces to four-dimensional RH-regular summability methods.

Using the above results, Patterson and Savaş [5] presented the following multidimensional version of Pruitt's result [8].

Theorem 2.2. *A necessary and sufficient condition for* $Y_{m,n} = \bar{Y}_m \bar{\bar{Y}}_n$ *to* P*-converge to* μ *in probability is that* $\max_{k,l} |a_{m,n,k,l}| = \max_{k,l} |a_{m,k} a_{n,l}|$ *converges to* 0 *in the Pringsheim sense.*

3. The main results

We begin the main section with the following RH-conservative characterization of P-convergence in probability.

Theorem 3.1. *Let* $(X_{k,l}) = (\bar{X}_k \bar{\bar{X}}_l)$ *be a double sequence of non-degenerate independently identically distributed random variables such that* $E(X_{k,l}) = \mu$ *and* $E(X_{k,l}) < \infty$ *for each* (k, l)*. Suppose that* $A = (a_{m,n,k,l})$ *is an RH-conservative matrix; then the necessary and sufficient condition for* $Y_{m,n}$ *to* P*-converge in probability to* $\mu(a - \sum_{k,l} c_{k,l}) + \sum_{k,l} c_{k,l} X_{k,l}$ *is that*

$$\mathrm{P} - \lim_{m,n} \sup_{k,l} |a_{m,n,k,l} - c_{k,l}| = 0. \tag{3.1}$$

Proof. Let X be the factorable random variable such that $F = \bar{F}\bar{\bar{F}}$ is the common factorable distribution function of $\bar{X}$ and the $\bar{\bar{X}}$'s. Let

$$\|A\| = \sup_{m,n} \sum_{k,l} |a_{m,n,k,l}| < \infty.$$

The RH-C conditions grant us the following:

$$E\left(\sum_{k,l} |a_{m,n,k,l} X_{k,l}|\right) \leq \|A\| E(|X_{k,l}|) < \infty,$$