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Asymptotic synchronization of the Colpitts oscillator

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ABSTRACT

In this paper we deal with the observer-based asymptotic synchronization problem for a class of chaotic oscillators. Some results based on a differential algebraic approach are used in order to determine the algebraic observability of unknown variables. The strategy consists of proposing a slave system (observer) which tends to follow asymptotically the master system. The methodology is tested in the real-time asymptotic synchronization of the Colpitts oscillator by means of a proportional reduced order observer (PROO) of free-model type.

1. Introduction

Chaotic systems synchronization has been investigated since its introduction in the paper [1]. Among the publications dedicated to chaos synchronization, many different approaches can be found [2–4]. We cite the papers [4–7] which propose the use of state observers to synchronize chaotic systems; in references [8–10] use feedback controllers; in [11,12] use nonlinear backstepping control; in papers [13,14] consider synchronization time delayed systems; in works [15,16] consider directional and bidirectional linear coupling; papers [17,18] use nonlinear control; in [10,19] use adaptive control; in [20] apply sliding-mode techniques; in [21] attack the anti-synchronization problem; in [22] a fuzzy sliding-mode controller is applied; in [23] the synchronization in time delay systems is given, and so on.

Synchronization of the chaotic systems problem has received a great deal of attention among scientists in many fields due to its potential applications, such as: secure communications, biological systems, chemical reactions, etc., [4,8,24–26].

As we can note, there exist several methods to solve the synchronization problem since from the control theory perspective in this work, we study the asymptotic synchronization by means of state observers.

The method is based on a *master–slave* configuration [1]. The main characteristic is that the coupling signal is unidirectional, that is, the signal is transmitted from the master system (transmitter) to the slave system (receiver), the receiver is requested to recover the unknown (or full) state trajectories of the transmitter. By this fact, the terminology *transmitter–receiver* is also used. Thus, the chaos synchronization problem can be regarded as an observer design procedure, where the coupling signal is viewed as an output and the slave system is the observer [27–29].

The problem of observer design naturally arises in a system approach, as soon as one needs unmeasured internal information from external measurements. In general, it is clear that one cannot use as many sensors as signals of interest characterizing the system behavior for technological constraints, cost reasons, and so on, especially since such signals can come in a quite large number, and they can be of various types: they typically include parameters, time-varying signals characterizing the system (state variables), and unmeasured external disturbances [30,31].

As we know, it is almost impossible to measure all the elements of the state vector in practice (e.g., the unknown state variables, fault signals, etc.). Here arises a basic practical question: would it be possible to reconstruct these unknown

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signals? We give an answer to this question by introducing a basic definition related with the estimation (reconstruction) of the states, the algebraic observability property (AOP).

In this work the observability property for a class of systems (oscillators, chaotic systems, and systems with bounded dynamics) is determined by means of a relatively new approach which is related with the differential-algebra framework. This mathematical approach has been recently shown to be a very effective tool for understanding basic questions such as input–output inversions and observer realizations.

The main contributions consist of the following. (i) An observer as a numerical technique to reconstruct unknown variables is designed. Before proposing the observer structure we should verify whether the signal to be estimated satisfies the AOP. Then we design a PROO which is based on the free-model approach. The main advantage of the PROO is that the free-model quality of its structure allows us to reconstruct the unknown variables in spite of model uncertainties. (ii) The suggested approach is implemented in the real-time asymptotic synchronization of the Colpitts oscillator via the PROO.

The paper is organized as follows. In Section 2 the synchronization problem and its solution by means of a reduced order observer are treated. Section 3 presents the procedure for the synchronization in real-time of the Colpitts oscillator [32]. Section 4 illustrates the obtained experimental results and shows the performance of the reduced order observer. Finally, in Section 5 we close the paper with some concluding remarks.

2. Observer design

Let us consider the following nonlinear system:

$$\begin{aligned} \dot{x}(t) &= f(x, u) \\ y(t) &= h(\bar{x}) \end{aligned} \tag{1}$$

where $f \in \mathbb{R}^n$ is differentiable and satisfies $f(0, 0) = 0$, $x = (x_1, x_2, \dots, x_n)^T \in \mathbb{R}^n$ is the state vector, $\bar{x} \in \mathbb{R}^p$ represents the known states (with $1 \leq p < n$), $y \in \mathbb{R}^q$ denotes the output of the system, $h : \mathbb{R}^p \to \mathbb{R}^q$ is a continuous function and $u \in \mathbb{R}^l$ is the known input ($l \leq n$).

Now, let us consider the nonlinear system described by (1). We separate (1) into two dynamical systems with states $\bar{x} \in \mathbb{R}^p$ and $\eta \in \mathbb{R}^{n-p}$ respectively with $x^T = (\bar{x}^T, \eta^T)$. The first system describes the known states and the second represents unknown states, then the system (1) can be rewritten as:

$$\begin{aligned} \dot{x}(t) &= \bar{f}(x, u) \\ \dot{\eta}(t) &= \Delta(x, u) \\ y(t) &= h(\bar{x}) \end{aligned} \tag{2}$$

where $\bar{f} \in \mathbb{R}^p, f^T(x) = \left(\bar{f}^T(x, u), \Delta^T(x, u)\right)$, and $\Delta \in \mathbb{R}^{n-p}$ is an uncertain function. The problem is to estimate the state variable $\eta(t) = (\eta_{(p+1)}, \dots, \eta_n)^T$.[1] In order to solve this observation problem let us introduce the following property.

Definition 1 (*Algebraic Observability Property-AOP*)**.** Let us consider the nonlinear dynamical system (2). A state variable $\eta_i \in \mathbb{R}$ is said to be algebraically observable if it is algebraic over $\mathbb{R}\langle u, y\rangle$,[2] that is to say, η_i satisfies a differential algebraic polynomial in terms of $\{u, y\}$ and some of their first $r_1, r_2 \in \mathbb{N}$ time derivatives, respectively, i.e.,

$$\eta_i = \phi_i(u, \dot{u}, \dots, \overset{(r_1)}{u}, y, \dot{y}, \dots, \overset{(r_2)}{y}), \quad i \in \{p+1, \dots, n\}, \tag{3}$$

where $\phi_i : \mathbb{R}^{(r_1+1)l} \times \mathbb{R}^{(r_2+1)q} \to \mathbb{R}$.

If any unknown variable satisfies the AOP, then a numerical technique from the so-called observer can be used to reconstruct the required signal.

The next system represents the dynamics of the unknown states:

$$\dot{\eta}_i(t) = \Delta_i(x, u). \tag{4}$$

Lemma 1. *If the following hypotheses are satisfied:*

H1: *$\eta_i(t)$ satisfies the AOP (Definition 1), for $i \in \{p+1, \dots, n\}$.*
H2: *γ_i is a C^1 real-valued function.*
H3: *Δ_i is bounded, i.e., $|\Delta| \leq M < \infty$.*
H4: *For t_0, sufficiently large, there exists $K_i > 0$, such that, $\limsup_{t\to t_0} \frac{M}{K_i} = 0$.*

Then, the system

$$\dot{\hat{\eta}}_i = K_i(\eta_i - \hat{\eta}_i) \tag{5}$$

is an asymptotic reduced order observer of free-model type for system (4), where $\hat{\eta}_i$ denotes the estimate of η_i and $K_i \in \mathbb{R}^+$ determines the desired convergence rate of the observer.

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1 In practice, identification of the variable η depends on the variable choice to be estimated.

2 $\mathbb{R}\langle u, y\rangle$ denotes the differential field generated by the field $\mathbb{R}$, the input u, the measurable output y, and the time derivatives of u and y.