Computers and Mathematics with Applications 69 (2015) 675–687

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Efficient algorithms for hybrid regularizers based image denoising and deblurring

Xinwu Liu

School of Mathematics and Computational Science, Hunan University of Science and Technology, Xiangtan, Hunan 411201, China

ARTICLE INFO

Article history:
Received 22 June 2014
Received in revised form 5 February 2015
Accepted 8 February 2015
Available online 2 March 2015

Keywords:
Image denoising
Image deblurring
Hybrid regularizers
Relaxation method
Alternating minimization algorithm

ABSTRACT

To better eliminate the staircase effect and simultaneously preserve edge details, this paper investigates a hybrid regularizers model for image denoising and deblurring. This technique closely incorporates the advantages of the classical total variation (TV) filter and the fourth-order filter. Computationally, we develop an extremely efficient relaxation scheme and alternating minimization algorithm, and give the rigorous convergence analyses there in detail. Provided experimental results distinctly illustrate the high efficiency of the addressed numerical algorithms. Also, in comparison with the recovered results by the state-of-the-art models, simulations manifestly demonstrate the competitive performance of our proposed scheme in reducing the blocky images and sharply maintaining the edge features.

1. Introduction

The problems of image denoising and deblurring occupy prominent roles in image processing and computer vision. One remarkable example of variational models for image denoising is the Rudin–Osher–Fatemi (ROF) model, involving the TV norm as an edge-preserving regularization functional and initially introduced in [1] as

$$\min_u \int_\Omega |Du| + \frac{\lambda}{2}\int_\Omega (u-f)^2 \mathrm{d}x, \tag{1.1}$$

where $\Omega \subseteq \mathbb{R}^n$ is an open, bounded Lipschitz domain, D denotes the gradient operator, $\lambda > 0$ stands for the penalty parameter, u and f represent the unknown true image and the observed one, respectively. Computationally, the corresponding partial differential equation (PDE) of (1.1) is a second-order equation. Furthermore, to better preserve important corners and junctions, the adaptive TV regularization based variational denoising model was developed by Strong and Chan [2,3] as

$$\min_u \int_\Omega g|Du| + \frac{\lambda}{2}\int_\Omega (u-f)^2 \mathrm{d}x, \tag{1.2}$$

where $g(x)$ is a spatially varying weighting coefficient. The usual choice for the diffusivity function given in [4] is $g(x) = \frac{1}{1+\mathcal{K}|\nabla G_\sigma * f|^2}$ for regulating the degree of smoothing adaptively, $\mathcal{K} > 0$ being a threshold parameter tuned for balancing the noise reduction and detail preservation capability, and $G_\sigma(x) = \frac{1}{2\pi\sigma^2}\exp\left(-\frac{|x|^2}{2\sigma^2}\right)$ being the Gaussian filter with standard deviation σ. In diffusion, the auxiliary function g is chosen as a decreasing function of $|\nabla G_\sigma * f|$ such that g is smaller near a likely edge and larger away from the possible region's boundaries. To sum up, the introduced edge-stopping function has the capability of effectively preventing the diffusion of the edges and sharply preserving the edges while removing noise.

E-mail address: lxinwu@163.com.

http://dx.doi.org/10.1016/j.camwa.2015.02.011

The models mentioned above are efficient for removing noise and simultaneously maintaining the sharp edges. Unfortunately, the numerous staircasing artifact emerges in the recovered image attributing to the TV regularization framework. To overcome this shortcoming, high-order PDE (typically, fourth-order PDE) filters, such as fourth-order anisotropic diffusion strategy [5–10], and fourth-order PDE regularization based minimization scheme [11–15] have been researched and applied for image restoration successfully. Thereinto, the classical Lysaker–Lundervold–Tai (LLT) model was originally formulated in [14] for image restoration in the following form

$$\min_u \int_\Omega |D^2u| + \frac{\lambda}{2}\int_\Omega (u-f)^2 \mathrm{d}x. \tag{1.3}$$

Here, the corresponding diffusion equation of (1.3) is a fourth-order PDE filter. Numerical experiments demonstrate that the LLT model outperforms the ROF model in terms of recovery of smooth regions. A flaw in an otherwise perfect thing is that, this fourth-order filter frequently leads to edge blurring when image denoising.

To reduce the staircase artifact and synchronously avoid the edges blurring, several efficient numerical strategies have been sprung up recently, such as the adaptive LLT filter [16], and the hybrid regularization model [17–19] combining the advantages of the ROF model and the LLT scheme, etc. Among which, the double regularizers weighted scheme displayed in [18] is characterized by

$$\min_u \int_\Omega (1-g)|Du| + \int_\Omega g|D^2u| + \frac{\lambda}{2}\int_\Omega (u-f)^2 \mathrm{d}x. \tag{1.4}$$

The related simulations indicate that the combination scheme performs better than the pure second-order and fourth-order PDE models. To our best knowledge, the steepest descent algorithm employed for solving (1.4) is generally very slow, owing to the nonlinearity and strict restrictions on the Courant–Friedrichs–Lewy (CFL) condition.

Furthermore, it is widely acknowledged that obtained data may be corrupted by multifarious blurs in the real world. Considering the blur effects, the corresponding image restoration can be implemented by solving the optimization problem

$$\min_u \int_\Omega (1-g)|Du| + \int_\Omega g|D^2u| + \frac{\lambda}{2}\int_\Omega (Hu-f)^2 \mathrm{d}x, \tag{1.5}$$

where $g(x)$ denotes the edge-stopping function defined as in (1.2), and H is a bounded linear blur operator, subject to $H \cdot 1 \neq 0$.

However, to further reduce the computational complexity for solving (1.5), our objective of the current article is to investigate its anisotropic hybrid regularization version, formulated as follows

$$\min_u \int_\Omega (1-g)(|\nabla_x u| + |\nabla_y u|)\mathrm{d}x + \int_\Omega g(|\Delta_x u| + |\Delta_y u|)\mathrm{d}x + \frac{\lambda}{2}\int_\Omega (Hu-f)^2 \mathrm{d}x. \tag{1.6}$$

Complementally, let $M \times N$ represent the image size, then the first order difference operators ∇_x and ∇_y can be defined as

$$\nabla_x u_{i,j} = \begin{cases} 0, & \text{if } i = 1, \\ u_{i,j} - u_{i-1,j}, & \text{if } 1 < i \le M, \end{cases} \tag{1.7}$$

$$\nabla_y u_{i,j} = \begin{cases} 0, & \text{if } j = 1, \\ u_{i,j} - u_{i,j-1}, & \text{if } 1 < j \le N, \end{cases} \tag{1.8}$$

and the second order difference operators Δ_x and Δ_y are characterized by

$$\Delta_x u_{i,j} = \begin{cases} u_{1,j} - u_{2,j}, & \text{if } i = 1, \\ 2u_{i,j} - u_{i-1,j} - u_{i+1,j}, & \text{if } 1 < i < M, \\ u_{M,j} - u_{M-1,j}, & \text{if } i = M, \end{cases} \tag{1.9}$$

$$\Delta_y u_{i,j} = \begin{cases} u_{i,1} - u_{i,2}, & \text{if } j = 1, \\ 2u_{i,j} - u_{i,j-1} - u_{i,j+1}, & \text{if } 1 < j < N, \\ u_{i,N} - u_{i,N-1}, & \text{if } j = N. \end{cases} \tag{1.10}$$

Our significant contributions of this paper can be generalized as follows. First of all, the proposed anisotropic hybrid regularizers weighted scheme, combining the merits of the TV regularization and fourth-order filter, substantially reduces the staircase artifact and simultaneously avoids the blurring effect. Secondly, the most important contribution is developing an extremely efficient relaxation algorithm and alternating minimization algorithm for image denoising and deblurring based on the variational framework (1.6). The proposed strategies avoid solving the complicated PDEs directly and accelerate the computational speed dramatically.

The organization of this paper is generalized as follows. In Section 2, we describe the necessary definitions and notions for the proposed model. Section 3 is dedicated to the description of the proposed efficient numerical algorithms in detail. And their rigorous convergence proofs are analyzed in Section 4. Numerical results aiming at demonstrating the proposed strategy are provided in Section 5. Finally, concluding remarks are generalized in Section 6.