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Wire coating analysis with Oldroyd 8-constant fluid by Optimal Homotopy Asymptotic Method

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ABSTRACT

In this study the wire coating in a pressure type die with the bath of Oldroyd 8-constant fluid with pressure gradient is investigated. The non-linear ordinary differential equation in dimensionless form is obtained, which is solved for the velocity profile using the Optimal Homotopy Asymptotic Method (OHAM). The effect of Dilatant constant α, the Psendoplastic constant β, and the pressure gradient on velocity distribution and shear stress is studied. Shear stress is examined under the effect of the viscosity parameter η_0. Moreover, the volume flow rate and average velocity is carefully studied with changing the domain (thickness) of the polymer and varying the parameter α, β and the pressure gradient.

1. Introduction

Wire coating is often used for the purpose of high and low voltage and protection against corrosion. The wire coating is performed by dragging the wire in a molten polymer inside the coating unit. Due to the shear stress between the wire and the molten polymer the wire is coated. The thickness of the coated wire is the same as the thickness of the die at the exit. A typical wire coating unit consists of a pay off device, preheater, extruder device with a cross head die, cooling device, and a take-up reel as shown in Fig. 1. The pay off device is a reel stand carrying a reel of uncoated wire. The preheater is used to give a temperature to the wire, while the extruder device fitted with a cross-head contains a canonical die. The cooling device is used for cooling the wire. The take-up reel is used for winding the coated wire on a rotating reel.

Wire coating is an important industrial process in which different types of polymer are used. The coating depends on the geometry of the die, the viscosity of the fluid, the temperature of the wire and the polymer used for coating the wire.

Akhter and Hashmi [1,2] have studied wire coating using power law fluid and have investigated the effect of the change in viscosity. Siddiqui et al. [3] studied wire coating extrusion in a pressure-type die in the flow of a third grade fluid. Fenner and Williams [4] carried out an analysis of the flow in the tapering section of a pressure type die. Sajjid et al. [5] studied the wire coating with Oldroyd 8-constant fluid without pressure gradient using the Homotopy Analyses Method (HAM), and give the solution for the velocity field in the form of a series.

We investigate the Oldroyd 8-constant fluid flow under pressure and examine carefully the velocity distribution, shear stress, volume flow rate, average velocity and the effect of velocity distribution while, changing the thickness of fluid under the same geometry with the Optimal Homotopy Asymptotic Method (OHAM) and obtained satisfactory results. The effect of Dilatant constant α, the Psendoplastic constant β, and the pressure gradient on velocity distribution and shear stress is studied. Shear stress is also examined by changing the viscosity parameter η_0. Here, we use a new homotopy approach,

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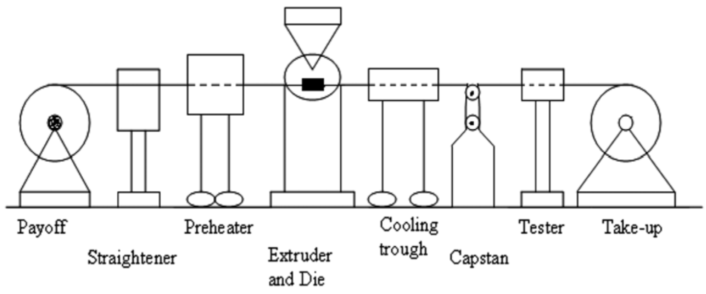

Fig. 1. Typical wire coating process.

namely OHAM to solve the nonlinear differential equation. Marinca and Herişanu [6–8] proposed this homotopy technique called the Optimal Homotopy Asymptotic Method (OHAM) and it proved to be a reliable approach to strongly nonlinear problems. In a series of papers by Marinca and Herişanu [9–11] and Islam et al. [12,13] have shown that this method is a more powerful tool than other perturbation tools for nonlinear problems.

2. Basic equation

Basic equations which govern the flow of an incompressible fluid neglecting the thermal effects are:

$$\nabla\cdot\underline{u}=0, \tag{1}$$

$$\rho\frac{D\underline{u}}{Dt}=\mathrm{div}\underline{T}+\rho\underline{f}, \tag{2}$$

where $\underline{u}$ is the velocity vector of the fluid, $\underline{T}$ is the Cauchy stress tensor, ρ is the constant density, $\underline{f}$ is the body force per unit mass and $\frac{D}{Dt}$ is the material derivative.

The Rheological equation of state for an Oldroyd 8-constant model is given by

$$\underline{T}=-P\underline{I}+\underline{S}, \tag{3}$$

where P denotes the pressure, $\underline{I}$ is the identity unit tensor and the extra stress tensor $\underline{S}$ is defined as

$$\begin{aligned}&\underline{S}+\lambda_1\overset{\nabla}{\underline{S}}+\frac{1}{2}(\lambda_1-\mu_1)\left(\underline{A}_1\,\underline{S}+\underline{S}\,\underline{A}_1\right)+\frac{1}{2}\mu_0\left(\mathrm{tr}\,\underline{S}\right)\underline{A}_1+\frac{1}{2}\upsilon_1\left(\mathrm{tr}\,\underline{S}\,\underline{A}_1\right)\underline{I}\\&=\eta_0\left(\underline{A}_1+\lambda_2\overset{\nabla}{\underline{A}}_1+(\lambda_2-\mu_2)\underline{A}_1^2+\frac{1}{2}\upsilon_2\left(\mathrm{tr}\,\underline{A}_1^2\right)\underline{I}\right).\end{aligned} \tag{4}$$

Here, the constants $\eta_0, \lambda_1, \lambda_2$ are respectively, zero shear viscosity, relaxation time and retardation time. The other five constants $\mu_0, \mu_1, \mu_2, \upsilon_1, \upsilon_2$ are associated with non-linear terms.

The upper contra-variant convected derivative designed by ∇ over $\underline{S}$ and $\underline{A}_1$ is defined as follows

$$\overset{\nabla}{\underline{S}}=\frac{D\underline{S}}{Dt}-\left[\left(\nabla\underline{u}\right)^T\underline{S}+\underline{S}\left(\nabla\underline{u}\right)\right] \tag{5}$$

$$\overset{\nabla}{\underline{A}}_1=\frac{D\underline{A}_1}{Dt}-\left[\left(\nabla\underline{u}\right)^T\underline{A}_1+\underline{A}_1\left(\nabla\underline{u}\right)\right] \tag{6}$$

$$\text{where } \underline{A}_1=\left(\nabla\underline{u}\right)+\left(\nabla\underline{u}\right)^T \text{ and } \frac{D\underline{S}}{Dt}=\left[\frac{\partial}{\partial t}+\left(\underline{u}\cdot\nabla\right)\right]\underline{S}. \tag{7}$$

It should be noted that the model (4) includes as special cases the following

(i) If $\eta_0=\lambda_1=\lambda_2=\mu_0=\mu_1=\mu_2=\upsilon_1=\upsilon_2=0$, we recover the Newtonian model.
(ii) If $\eta_0=\lambda_1=\mu_0=\mu_1=\mu_2=\upsilon_1=\upsilon_2=0$ and $\lambda_2=\lambda_2$, the second grade fluid model is obtained.
(iii) If $\eta_0=\lambda_2=\mu_0=\mu_2=\upsilon_1=\upsilon_2=0$ and $\lambda_1=\lambda_1$, $\mu_1=\lambda_1$ then the upper convected Maxwell model is recovered.
(iv) If $\lambda_1=\lambda_1$ and $\eta_0=\lambda_2=\mu_0=\mu_1=\mu_2=\upsilon_1=\upsilon_2=0$, we reach the co-rotational Maxwell model.
(v) If $\lambda_1=\lambda_1$, $\lambda_2=\lambda_2$, $\mu_1=\lambda_1$, $\mu_2=\lambda_2$, $\eta_0=\eta_0$ and $\upsilon_1=\upsilon_2=0$ then the Oldroyd 4-constant model is recovered.
(vi) If $\lambda_1=\lambda_1$, $\lambda_2=\lambda_2$, $\mu_1=\lambda_1$, $\mu_2=\lambda_2$, and $\eta_0=\upsilon_1=\upsilon_2=0$, we arrive at the upper convected Jeffery (Oldroyd *B*-model).
(vii) If $\lambda_1=\lambda_1$, $\lambda_2=\lambda_2$ and $\eta_0=\mu_0=\mu_1=\mu_2=\upsilon_1=\upsilon_2=0$, we gain the co-rotational Jeffery model.