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Velocity–conductivity relationships for mantle mineral assemblages in Archean cratonic lithosphere based on a review of laboratory data and Hashin–Shtrikman extremal bounds

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ABSTRACT

Can mineral physics and mixing theories explain field observations of seismic velocity and electrical conductivity, and is there an advantage to combining seismological and electromagnetic techniques? These two questions are at the heart of this paper. Using phenomenologically-derived state equations for individual minerals coupled with multi-phase, Hashin–Shtrikman extremal-bound theory we derive the likely shear and compressional velocities and electrical conductivity at three depths, 100 km, 150 km and 200 km, beneath the central part of the Slave craton and beneath the Kimberley region of the Kaapvaal craton based on known petrologically-observed mineral abundances and magnesium numbers, combined with estimates of temperatures and pressures. We demonstrate that there are measurable differences between the physical properties of the two lithospheres for the upper depths, primarily due to the different ambient temperature, but that differences in velocity are negligibly small at 200 km. We also show that there is an advantage to combining seismic and electromagnetic data, given that conductivity is exponentially dependent on temperature whereas the shear and bulk moduli have only a linear dependence in cratonic lithospheric rocks. Focussing on a known discontinuity between harzburgite-dominated and lherzolitic mantle in the Slave craton at a depth of about 160 km, we demonstrate that the amplitude of compressional (P) wave to shear (S) wave conversions would be very weak, and so explanations for the seismological (receiver function) observations must either appeal to effects we have not considered, or imply that the laboratory data require further refinement.

1. Introduction

Petrological physical-property measurements in the laboratory connect geophysical observations to subsurface materials and processes. Most of the measurements are made on single grains of constituent minerals that form the lithosphere, and from those measurements equations are derived that best fit the data and show the dependence of various parameters, such as the bulk and shear moduli and electrical conductivity, on temperature, pressure, grain size, etc. Once those single grain equations are known, the next task is to determine the likely physical parameters of whole rocks using various mixing theories and relationships. Our approach here is to use extremal-bound theory for those mixing relationships, rather than the Voigt–Reuss–Hill (Voigt, 1928; Reuss, 1929; Hill, 1952) estimates more routinely applied in seismology.

We choose to be very selective in the type of rocks we wish to numerically manufacture, and restrict ourselves to dry, cratonic mantle which can be described in simple mineralogical and physical terms. Although our selection suffers from not having the breadth of Hacker et al.'s (2003) consideration of the multitude of minerals in a subduction zone setting, we consider each mineral phase in great detail, and for both elastic moduli and electrical conductivity.

We construct typical continental lithospheric mantle "rocks" based on assemblages of four minerals, namely olivine, orthopyroxene, clinopyroxene and garnet, from known compositions at three depths, 100 km, 150 km and 200 km, below the Lac de Gras region of Slave craton (northern Canada) and below the Kimberley region of the Kaapvaal craton (South Africa). We choose these two regions as they are, by far, the best known geochemically and petrologically in the world. Furthermore, the two are clearly different in their chemical compositions (Stachel et al., 2003), and we wish to determine whether these chemical differences translate into measurable physical differences in velocity and conductivity. In particular, the central Slave craton displays a strong chemical stratification, with a high Mg# layer comprising 60% harzburgite (Hz) and 40% lherzolite (Lh) (Griffin et al., 1999; Menzies et al., 2004) to about 160 km (Menzies et al., 2004), overlying a more fertile, low Mg#, dominantly lherzolitic layer to a depth of around 200 km (Griffin et al., 1999; Menzies et al., 2004). In the upper layer 20–40% of the Hz is low-CaO Hz, evidence of the strong

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depletion of this layer. In contrast, the Kimberley region of the Kaapvaal craton exhibits far more uniform properties without strong chemical layering, and with "normal" lherzolitic rocks throughout.

Our approach is similar to that of others, particularly Hacker et al. (2003), with their published Excel spreadsheet for a broader range of minerals (Hacker and Abers, 2004). We agree with the comment in Hacker et al. (2005) to Bosquet et al.'s (2005) criticism of their 2003 paper that this approach has the advantage that it is grounded in reality—known laboratory-determined petrophysical properties of minerals coupled with known mineral abundances and, in our case, a conservative mixing law that includes both grain and surface effects.

One important point in our analysis is that we exclude any "exotic" minor phases. These have less effect on seismic velocities, but some phases, such as carbon in graphite form or lining grain boundaries, have a considerable effect on electrical conductivity (e.g., Jones et al., 2003; Jones and Craven, 2004). Some support for the role of graphite in enhancing mantle conductivity comes from the depth extent of the Central Slave Mantle Conductor that appears to be limited to above the graphite–diamond stability field (Jones et al., 2003).

Kopylova et al. (2004) undertook a somewhat similar exercise for seismic velocities for the northern Slave and the southern Slave, but not for the central Slave which exhibits the strong petrological stratification. The mixing rule they adopted was a simple weighted arithmetic average of the velocities for the constituent minerals, rather than the formal extremal bound rules that we use here. Similarly, Bagdassarov et al. (2007) constructed electrical conductivity profiles from laboratory determinations of conductivity on minerals from xenoliths recovered on the Slave craton, but used a simple logarithmic averaging scheme for their mixing rule.

Having derived the velocities and conductivity at various depths beneath the Slave and Kaapvaal craton, we construct cross-plots to try to identify interdependence between the moduli (velocity) and conductivity. This is the reverse of the approach of Gibiansky and Torquato (1993, 1996) and Carcione et al. (2007) who defined formal relationships between moduli and conductivity for two-phase composites.

We consider separately the effects of varying each of the controlling conditions—temperature, iron content and other aspects of the chemical composition/mineralogy—and demonstrate that temperature has the greatest effect on the bulk physical properties.

Finally, we determine the parameters on either side of the harzburgite/lherzolite boundary at 160 km beneath the central part of the Slave craton. We demonstrate that, according to the best available mineral physics data and the most valid mixing theories, we should not be able to observe this boundary with teleseismic receiver functions, whereas one is clearly seen (Snyder et al., 2004). Thus we conclude that either the mineral physics data require refinement, and/or that there is an inadequately known scaling from the laboratory scale to the field scale, and/or that the mixing laws are inappropriate.

2. Seismic velocity of mantle minerals

2.1. Estimates of the bulk and shear moduli of mantle minerals

The bulk (K) and shear (G) moduli and density (ρ) of the dominant mantle minerals olivine (Ol), orthopyroxene (Opx), clinopyroxene (Cpx) and garnet (Gt) have been measured by many laboratories over more than three decades, and Table 1 lists recently-reported values. For some of them the standard temperature and pressure (25 °C and 1 atm.) values are given, and there are noted variations of the moduli with pressure, temperature and magnesium number (Mg#, Mg/(Mg+Fe), usually multiplied by 100).

The variation of the moduli with pressure and temperature has also been reported by various laboratories, and example values are given in Table 1, some of which represent summaries themselves. There is clearly a wide range of reported values for the moduli, and their dependence on Mg#, temperature and pressure. For the purposes of our study, we take the formulae of James et al. (2004) for olivine, orthopyroxene and garnet, and the formulae of Goes et al (2000) for clinopyroxene. Goes et al.'s (2000) formulations yield virtually the same moduli as those in Isaak et al. (2006) for an Mg# of around 90, but include the pressure dependence for the bulk modulus

Table 1
Values of bulk and shear moduli and density from various authors, with pressure, temperature and Mg# dependence where available

Mineral	K_S (GPa)	$\delta K/\delta P$ ($\delta K^2/\delta P^2$)	$\delta K/\delta T$ (MPa/K)	G (GPa)	$\delta G/\delta P$	$\delta G/\delta T$ (MPa/K)	ρ (g/cm^3)	Reference
Ol	–	–	−17.5	–	–	−13.7		I93
Ol	129.0	–	−16	82−30.0*f*	–	−14	3.222	GGV00
Ol	129.0	–	−19	–	–	–	3.222 +1.82*f*	LL06
Ol	**128.6 +7.0*f***	**4.4 −2.0*f***	**−18.2 −0.09*f***	**79.1 −35.8*f***	**1.71 −1.23*f***	**−14.0 −0.18*f***	3.22 +1.32*f*	**JBSBC04**
Opx	102	10.9 (−1.6)	–	74.9	1.6	–	3.180	FLL98
Opx	111 −10.0*f*	–	−12	81 −29*f*	–	−11	3.198	GGV00
Opx	108.5	–	−26.3	77.9	–	−13.6	3.196	JSB07
Opx	102.5	–	–	74.2	–	–		PNSB07
Opx	114	–	−13	74	–	−11	3.204 +0.799*f*	LL07
Opx	**106.5 −5.2*f***	**11.0 −2.56*f***	**−26.8**	**75.0**	**1.6**	**−12.0**		**JBSBC04**
Cpx	117.2	–	–	72.2	–	–	3.327	CB98
Cpx	**105 +13*f***	**–**	**−13**	**67 −6*f***	**–**	**−10**	3.280	**GGV00**
Cpx	116.5	–	−12.3	72.8	–	−9.98		IOL06
Cpx	117.6	6.4	–	–	–	–		NBTO05
Cpx	117	–	−15	67	–	−14	3.277 +0.38*f*	LL07
Gt	154.5	4.71	–	89.7	4.71	–		JSSD04
Gt	173 +7*f*	–	−21	92 −7*f*	–	−10		GGV00
Gt	171	–	−16	91	–	−10	3.565 +0.76*f*	LL07
Gt	**171.2**	**4.9**	**−19.8**	**93.0**	**1.56**	**−10.0**		**JBSBC04**

The boldfaced formulae are the ones adopted herein.
I93: Isaak (1993); CB98: Collins and Brown (1998); FLL98: Flesch et al. (1998); GGV00: Goes et al. (2000); IOL06: Isaak et al. (2006); JBSBC04: James et al. (2004); JSSD04: Jiang et al. (2004); NBTO05: Nestola et al. (2004); PNSB07: LL06: Liu and Li (2006); Perrillat et al. (2007); JSB07: Jackson et al. (2007); f=iron number (X_{Fe})=(Fe/(Fe+Mg))=1−Mg#/100.