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A fast multiphase image segmentation model for gray images[☆]

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ABSTRACT

In this paper we propose a fast multiphase image segmentation model for gray images. Our proposed model is based on the piecewise constant multiphase Vese–Chan model, the globally convex segmentation method and the split Bregman method. Our proposed model has the advantages of the Vese–Chan model and can deal with piecewise constant multiphase images well. Besides, our proposed model is much more efficient than the Vese–Chan model by applying the globally convex segmentation method and the split Bregman method. We have tested our proposed model with many synthetic and real images. Experimental results have demonstrated the efficiency and robustness of our proposed model.

1. Introduction

Image segmentation [1–10] is a fundamental task in image processing and computer vision. Active contour models [1,6,11–14] have been widely used in image segmentation with promising results. One well-known active contour model is the Chan–Vese (CV) model [1] proposed by Chan and Vese. In [1], Chan and Vese presented the two phase level set formulation without using the image gradient. The CV model has been successful for images with two regions, each having a distinct mean of pixel intensity.

In order to segment images with multiple regions, many multiphase image segmentation models have already been proposed in [2–4]. For example, in [2] Zhao et al. presented a model which is devoted to motion of junctions and boundaries of multiple phases in a variational level set approach. Samson et al. applied the multiphase level set formulation from Zhao et al. to the reduced Mumford and Shah model for piecewise constant image segmentation in [3]. However, because each phase is represented via one level set function, these models have the problems of vacuum and overlap.

To overcome these problems, Vese and Chan extended the CV model to a multiphase level set formulation called the Vese–Chan (VC) model in [6]. The VC model is superior to the above mentioned multiphase models in [2–4] in the following aspects. First of all, it can avoid the vacuum and overlap problems automatically. Besides, fewer level set functions are needed to represent the same number of regions. Furthermore, it can represent boundaries with complex topologies. Although with these advantages, the VC model has one drawback that this model is not convex and local minima exist. Thus the initial

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guess becomes critical to get satisfactory results. The CV model, the two-phase formulation of the VC model also has this deficiency.

In [15], Chan et al. proposed the globally convex segmentation method to eliminate difficulties associated with the non-convexity. This globally convex segmentation method is much easier to handle numerically. Besides, it does not get stuck in local minima, and thus is more reliable. Chan et al. presented the globally convex CV model by solving a convex constrained minimization problem, which has the coincident stationary solution with the minimization problem of the CV model. Then Goldstein et al. applied the split Bregman method to the globally convex CV model and gave a fast numerical scheme for computing the minimizer in [16]. However, the globally convex CV model is still for images with two regions and cannot handle images with multiple regions.

In this paper we present a fast multiphase image segmentation model for gray images based on the piecewise constant VC model. We first define a new energy functional based on the piecewise constant VC model [6] and the globally convex image segmentation method. Then the new energy functional is modified by incorporating information from the edge with a non-negative edge detector function [13,17]. We then apply the split Bregman method for a fast minimization of the proposed energy functional. Finally, we apply our model to many synthetic and real gray images. Experimental results show that our model has advantages of the original VC model but is much more efficient. The robustness of our model to noises has also been demonstrated by experimental results. Actually, in our previous work [18] we have presented a fast multiphase image segmentation model for color images. However, that model for color images is indeed based on the proposed model in this paper. Logically, this paper should be published before the work [18], but it is delayed for some reason. In Section 2.1 of [18], we have briefly introduced our four-phase model for gray images but not detailed it. In this paper we will present the proposed model in detail. Besides, we will also supplement several important proofs missed in our previous work [18] in Section 3.2 and give a more delicate algorithm for our model in Section 4.1.

The remainder of this paper is organized as follows. The related piecewise constant VC model and the Bregman iteration are introduced briefly in Section 2. Our proposed model is presented in Section 3. The numerical implementation and experimental results of our proposed model are given in Section 4. This paper is concluded in Section 5.

2. Preparations

In this section we briefly review the related piecewise constant VC model and the Bregman iteration, which we will use in Section 3 for presenting our new model.

2.1. The piecewise constant VC model

Vese and Chan proposed the VC model for multiphase image segmentation in [6], which is the generalization of the CV model. Vese and Chan presented the VC model in two formulations, the piecewise constant formulation and piecewise smooth formulation. In this paper, we mainly focus on the piecewise constant formulation. Therefore, when we mention the VC model, we mean the piecewise constant VC model. In this section we will briefly review the piecewise constant VC model.

Let $\Omega \subset \mathbb{R}^2$ be the image domain and $I_0 : \Omega \to \mathbb{R}$ be a given gray image. Vese and Chan use $m = \log_2 n$ level set functions $\psi_i : \Omega \to \mathbb{R}$ $(1 \le i \le m)$ to represent n phases. They label the phases by J, with $1 \le J \le 2^m = n$. They introduce a constant vector of averages $\mathbf{c} = (c_1, \ldots, c_n)$ where $c_J = mean(I_0)$ in the phase J. They propose to minimize the following energy functional:

$$F_n^{VC}(\mathbf{c}, \boldsymbol{\Psi}) = \sum_{1 \le J \le n = 2^m} \int_\Omega (I_0 - c_J)^2 \chi_J d\mathbf{x} + \sum_{1 \le i \le m} \nu \int_\Omega |\nabla H(\psi_i)|, \tag{1}$$

where $\boldsymbol{\Psi} = (\psi_1, \ldots, \psi_m)$ is the vector level set function and H is the Heaviside function [1,6]. ν is a positive constant. χ_J is the characteristic function for each phase J defined as:

$$\chi_J(\mathbf{x}) = \begin{cases} 1, & \text{if } \mathbf{x} \in \text{phase } J \\ 0, & \text{else.} \end{cases} \tag{2}$$

In this paper we mainly focus on the four-phase image segmentation, thus we specify the four-phase VC energy functional. For a special case when $n = 4$, $m = 2$ level set functions are needed to represent 4 phases. Then the above energy functional $F_n^{VC}(\mathbf{c}, \boldsymbol{\Psi})$ can be rewritten as follows:

$$\begin{aligned} F_4^{VC}(\mathbf{c}, \boldsymbol{\Psi}) = & \int_\Omega (I_0 - c_1)^2 H(\psi_1) H(\psi_2) d\mathbf{x} + \int_\Omega (I_0 - c_2)^2 H(\psi_1)(1 - H(\psi_2)) d\mathbf{x} \\ & + \int_\Omega (I_0 - c_3)^2 (1 - H(\psi_1)) H(\psi_2) d\mathbf{x} + \int_\Omega (I_0 - c_4)^2 (1 - H(\psi_1))(1 - H(\psi_2)) d\mathbf{x} \\ & + \nu \int_\Omega |\nabla H(\psi_1)| + \nu \int_\Omega |\nabla H(\psi_2)|, \end{aligned} \tag{3}$$

where $\mathbf{c} = (c_1, c_2, c_3, c_4)$ is a constant vector and $\boldsymbol{\Psi} = (\psi_1, \psi_2)$.