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New solitary wave solutions for two nonlinear evolution equations

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ABSTRACT

In this paper, multiple exp-function method is employed to investigate exact multiple wave solutions for $(2+1)$-dimensional potential Kadomtsev–Petviashvili equation and $(3+1)$-dimensional Jimbo–Miwa equation. Not only already known multiple wave solutions are recovered, but also several new or more general multiple wave solutions are obtained.

1. Introduction

Nonlinear evolution equation (NLEE) is a kind of crucial mathematical model to describe many nonlinear physical phenomena. Traveling wave solutions of these equations play an important role in nonlinear science, for these solutions may well reflect inner aspects of nonlinear problems and can be easily used in further applications. Many efficient methods to analyze exact solutions of NLEEs have been proposed in the past few decades, such as Darboux transformation method [1–3], Bäcklund transformation method [4,5], Hirota bilinear method [6–8], homogeneous balance method [9,10], tanh-function method [11–14], sech-function method [15], Jacobi elliptic function method [16–18], exp-function method [19,20] and many others. Even though there is no unified method that can be used for all types of NLEEs, all these mentioned approaches are proved to be powerful, reliable and effective in handling a wide variety of NLEEs.

Recently, Ma proposed a straightforward and systematic method, called multiple exp-function method [21], to obtain multiple wave solutions of NLEEs. This technique is rather heuristic and possesses significant features that make it promising for the determination of exact solutions for numerous NLEEs.

In the following, the procedure of multiple exp-function method is described and two nonlinear evolution equations are examined to generate their one-wave, two-wave and three-wave solutions. Illustrative examples include $(2+1)$-dimensional potential Kadomtsev–Petviashvili (PKP) equation and $(3+1)$-dimensional Jimbo–Miwa (JM) equation.

2. The basic idea of multiple exp-function method

Let us formulate the multiple exp-function method by focusing a scalar $(1+1)$-dimensional nonlinear evolution equation

$$F(x, t, u_t, u_x, \ldots) = 0, \tag{1}$$

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where $u = u(x, t)$ and F is a polynomial in u and its derivatives. This method also works for systems of NLEEs and high-dimensional ones.

Step 1. Defining solvable differential equations

Introducing a sequence of exponential variables $\eta_i = \eta_i(x, t),\ 1 \le i \le n$, which are defined as

$$\eta_i = c_i e^{\xi_i}, \qquad \xi_i = k_i x - \omega_i t, \quad 1 \le i \le n, \tag{2}$$

where $c_i,\ 1 \le i \le n$, are constants, $k_i,\ 1 \le i \le n$, are angular wave numbers and $\omega_i,\ 1 \le i \le n$, are wave frequencies. It is obvious that these new variables satisfy the following linear partial differential relations

$$\eta_{i,x} = k_i \eta_i, \qquad \eta_{i,t} = -\omega_i \eta_i, \quad 1 \le i \le n. \tag{3}$$

Each of the functions $\eta_i,\ 1 \le i \le n$, describing a single wave and a multiple wave solution will be a combination of all of those single waves.

Step 2. Transforming NLEE

Multiple exp-function method is very concise and efficient, on account of the assumption that multiple wave solutions can be expressed in the form

$$u(x, t) = \frac{p(\eta_1, \eta_2, \dots, \eta_n)}{q(\eta_1, \eta_2, \dots, \eta_n)}, \tag{4}$$

in which

$$p = \sum_{r,s=1}^{n} \sum_{i,j=0}^{M} p_{rs,ij} \eta_r^i \eta_s^j, \qquad q = \sum_{r,s=1}^{n} \sum_{i,j=0}^{N} q_{rs,ij} \eta_r^i \eta_s^j, \tag{5}$$

and $p_{rs,ij}$, $q_{rs,ij}$ are constants to be determined by Eq. (1). In this way, based on the differential relations in (3), all partial derivatives of u with respect to x and t can be expressed in terms of $\eta_i,\ 1 \le i \le n$, for example,

$$\begin{aligned} u_t &= \frac{q \sum_{i=1}^{n} p_{\eta_i} \eta_{i,t} - p \sum_{i=1}^{n} q_{\eta_i} \eta_{i,t}}{q^2} = \frac{-q \sum_{i=1}^{n} \omega_i p_{\eta_i} \eta_i + p \sum_{i=1}^{n} \omega_i q_{\eta_i} \eta_i}{q^2}, \\ u_x &= \frac{q \sum_{i=1}^{n} p_{\eta_i} \eta_{i,x} - p \sum_{i=1}^{n} q_{\eta_i} \eta_{i,x}}{q^2} = \frac{q \sum_{i=1}^{n} k_i p_{\eta_i} \eta_i - p \sum_{i=1}^{n} k_i q_{\eta_i} \eta_i}{q^2}, \end{aligned} \tag{6}$$

where p_{η_i} and q_{η_i} are partial derivatives of p and q with respect to η_i.

A rational function equation in new variables $\eta_i,\ 1 \le i \le n$,

$$G(x, t, \eta_1, \eta_2, \dots, \eta_n) = 0, \tag{7}$$

is generated by substituting those derivatives of u with respect to x and t into original equation (1).

Step 3. Solving algebraic systems

By setting the numerator of the rational function $G(x, t, \eta_1, \eta_2, \dots, \eta_n)$ to be zero, a system of algebraic equations on variables k_i, ω_i, $p_{rs,ij}$ and $q_{rs,ij}$ is established. Solving the obtained algebraic system, two polynomials p and q and the wave exponents $\xi_i,\ 1 \le i \le n$, are obtained. Hence the multiple wave solutions u read

$$u(x, t) = \frac{p(c_1 e^{k_1 x - \omega_1 t}, \dots, c_n e^{k_n x - \omega_n t})}{q(c_1 e^{k_1 x - \omega_1 t}, \dots, c_n e^{k_n x - \omega_n t})}. \tag{8}$$

3. Application of multiple exp-function method

In this section, exact multiple wave solutions are constructed to $(2 + 1)$-dimensional PKP equation and $(3 + 1)$-dimensional JM equation by the multiple exp-function method while attempting three cases of polynomial functions p and q, including one-wave, two-wave and three-wave solutions for each equation.

3.1. *The wave solutions to* $(2 + 1)$*-dimensional PKP equation*

The $(2 + 1)$-dimensional generalization of the Korteweg de Vries equation [22]

$$u_t + \frac{3}{4} u_x^2 + \frac{1}{4} u_{xxx} + \frac{3}{4} \int_0^x u_{yy} dx' = 0, \tag{9}$$