Computers and Mathematics with Applications 60 (2010) 1026–1037

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Utilization of SOMA and differential evolution for robust stabilization of chaotic Logistic equation

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ARTICLE INFO

Keywords:
Chaos
Control
Optimization
Evolutionary algorithms
Differential evolution
SOMA

ABSTRACT

This paper deals with the utilization of two evolutionary algorithms Self-Organizing Migrating Algorithm (SOMA) and Differential Evolution (DE) for the optimization of the control of chaos. This paper is aimed at an explanation on how to use evolutionary algorithms (EAs) and how to properly define the advanced targeting cost function (CF) securing fast, precise and mainly robust stabilization of selected chaotic system on a desired state for any initial conditions. The role of EA here is as a powerful tool for an optimal tuning of control technique input parameters. As a model of deterministic chaotic system, the one-dimensional discrete Logistic equation was used. The four canonical strategies of SOMA and six canonical strategies of DE were utilized. For each EA strategy, repeated simulations were conducted to outline the effectiveness and robustness of used method and targeting CF securing robust solution. Satisfactory results obtained by both heuristic and the two proposed cost functions are compared with previous research, given by different cost function designs.

1. Introduction

The question of targeting (faster stabilization) with application to chaos control has attracted researchers since the first method for controlling of chaos was developed. The several first approaches for targeting have used special versions of OGY (Ott–Grebogi–York) control scheme [1,2] or collecting information about trajectories, which falls close to desired state [3]. Later, numerous methods were based on adaptive approach [4], center manifold targeting [5] or neural networks [6,7].

Currently, evolutionary algorithms (EAs) [8–13] are known as powerful tools for almost any difficult and complex optimization problem. But the quality of obtained results through optimization mostly depends on proper design of the used cost function, especially when the EAs are used for optimization of chaos control. The results of numerous simulations lend weight to the argument that deterministic chaos in general and also any technique to control of chaos are sensitive to parameter setting, initial conditions and in the case of optimization, they are also extremely sensitive to the construction of used cost function.

This research utilized Pyragas' delayed feedback control technique ETDAS (Extended Time Delay Auto Synchronization) [14–16]. Unlike the original OGY control method [17], it can be simply considered as a targeting and stabilizing algorithm together in one package [18]. Another big advantage of the Pyragas method is the amount of accessible control parameters. This is very advantageous for successful use of optimization of parameter setting by means of EA, leading to improvement of system behavior and better and faster stabilization to the desired periodic orbits. Some research in this field has recently been done using EAs for optimization of local control of chaos [19,20], however our approach is different.

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doi:10.1016/j.camwa.2010.03.059

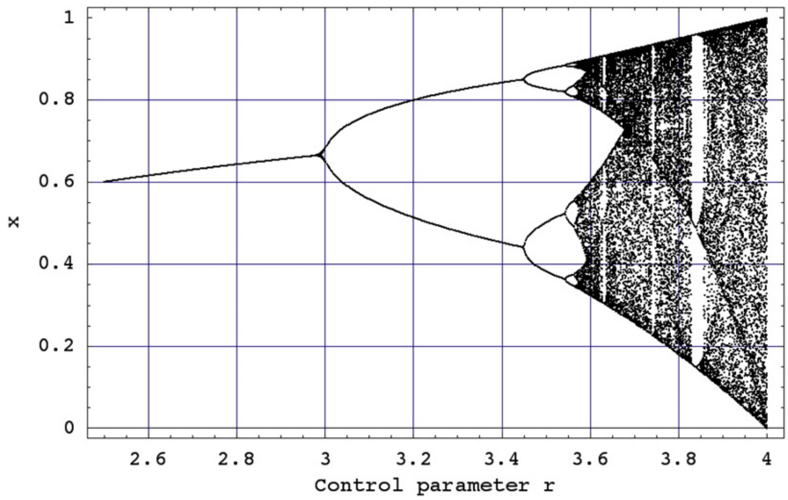

Fig. 1. Bifurcation diagram of Logistic equation.

The role of EA in this instance is as a powerful tool for an optimal tuning of control technique input parameters based on advanced targeting cost functions securing the stabilization to desired UPO (unstable periodic orbit) for any initial conditions. Numerous applications either of canonical DE or special version of DE [21–23] and SOMA [24] have proven that these heuristics are suitable for solving a difficult class of problems [25].

This work presents an accumulation of research [26] and also collates and elaborates the experiences with application of EA to chaos control [27,28] in order to reach better results and decrease the influence of negative phenomenon which can occur in such a challenging task, which is chaos control.

The main aim of this paper is not to show, which heuristic or its strategy is better or worse, but to test the selected EAs in such a challenging task, which is optimization of chaos control.

2. Problem design

2.1. Problem selection and case studies

The chosen example of a chaotic system was the one-dimensional Logistic equation as in form (1).

$$x_{n+1} = rx_n\left(1 - x_n\right). \quad (1)$$

The Logistic equation (Logistic map) is a one-dimensional discrete-time example of how a complex chaotic behavior can arise from very simple nonlinear dynamical equation. This chaotic system was introduced and popularized by the biologist Robert May [29]. It was originally introduced as a demographic model of a typical predator–prey relationship. The chaotic behavior can be observed by varying the parameter r. When $r = 3.57$, this is the beginning of chaos, at the end of the period-doubling behavior. When $r > 3.57$, the system exhibits chaotic behavior.

The example of this behavior can be clearly seen from the bifurcation diagram in Fig. 1.

This work primarily consists of three case studies. All of them are focused on an estimation of three accessible control parameters for ETDAS control method to stabilize desired UPO, and a comparison of obtained results for used cost function. Desired UPOs are the following: p-1 (a fixed point) in the first case, p-2 (higher periodic orbit—oscillation between 2 values) in the second case and p-4 (also high periodic orbit—oscillation between 4 values) in the last case. All simulations were 50 times repeated for each EA strategy. The control method—ETDAS in the discrete form suitable for Logistic equation has the form (2).

$$\begin{aligned} x_{n+1} &= rx_n\left(1 - x_n\right) + F_n \\ F_n &= K\left[\left(1 - R\right)S_{n-m} - x_n\right] \\ S_n &= x_n + RS_{n-m} \end{aligned} \quad (2)$$

where K and R are adjustable constants, F is the perturbation, S is given by a delay equation utilizing previous states of the system and m is the period of m-periodic orbit to be stabilized. The perturbation F_n in Eqs. (2) may have an arbitrarily large value, which can cause diverging of the system outside the interval {0, 1}. Therefore, F_n should have a value between, $-F_{max}$ and F_{max}, and EA should find an appropriate value of this limitation to avoid diverging of the system.