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A note on the Krone deposition equation and significance of floc aggregation

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ABSTRACT

For modeling the rate of deposition of cohesive flocs in estuaries the Krone equation is extensively used. It was derived from flume experiments on muddy sediment from the San Francisco Bay, and is applicable to low suspended sediment concentration environments in which shear-induced aggregation – the growth and break up of flocs – has a limited role. It is shown that the use of this equation can lead to erroneous estimates of the mass deposition flux at typically higher estuarine concentrations. Krone's own experimental data permit the development of a more general equation accounting for the effects of concentration and turbulent shear rate on aggregation. These effects are dramatically observed in a deposition test in which a wire mesh was inserted in the flow to change the turbulent shearing rate and increase deposition. Even with the inclusion of aggregation effect in the general equation, field-based observations from San Francisco Bay suggest that typical flumes generally may not meet the space and time scaling requirements for field application of laboratory data. Thus, although the Krone equation should be eschewed in favor of the general equation, interpretations of model-predicted deposition rate must not be accepted without robust field-based verification.

1. Introduction

The transport of cohesive sediment flocs in estuaries is, in general, strongly influenced by turbulent shear-induced aggregation including the growth and breakup of flocs, which in turn determine the erosion and deposition mass fluxes (Winterwerp et al., 2006; Manning and Dyer, 2007; Soulsby et al., 2013). Since the characteristically diffused bed–water boundary at which these fluxes occur is difficult to model, a relatively new approach has been to treat the boundary as an interface internal to modeled domain (e.g. Le Hir, 1997; Hsu et al., 2003). However, such modeling remains complex and is the reason simple analytic equations continue to be used. Among these, the most well-known formula is the “Krone equation” reported by Krone (1962) and Einstein and Krone (1962) for the time-rate of change of the mass concentration of suspended flocs as a function of bed shear stress. Recent studies (e.g. Winterwerp, 2007) have pointed out that, when used under the assumption of constant floc diameter and density in the tidal environment, the Krone equation (in tandem with an erosion equation) produces physically questionable results with respect to predicted tidal variation of the suspended sediment concentration. Yet, in general, the equation remains a popular choice in fine sediment transport models (e.g. Martin and McCutcheon, 1998).

The Krone equation was derived for low concentrations of suspended sediment using mud from the San Francisco Bay in a laboratory flume with non-oscillatory flow. The concentration C (dry mass per unit wet sample) can be conveniently expressed as solids volume fraction ϕ, where $\phi = C / \rho_s$ and the material density ρ_s is nominally equal to 2650 kg m^{-3}. When ϕ is below a limiting value ϕ_l, which is typically between 4×10^{-5} and 1×10^{-4} for the Bay sediment, aggregation by shear is slow because inter-particle collisions are infrequent. The floc diameter d_f and the settling velocity w_s are practically independent of ϕ as well as the shear rate G, i.e. the root-mean-square of the gradient in turbulent velocity fluctuations.

In estuaries including the San Francisco Bay, ϕ often exceeds ϕ_l, either with tidal periodicity or under storm waves depending on the location, and aggregation becomes increasingly important with increasing ϕ (Manning and Schoellhamer, 2013). Since in general the floc diameter and the settling velocity depend on ϕ as well as G, the accuracy of changes in the time-rate and spatial pattern of shoaling predicted by the Krone equation becomes tenuous.

Unfortunately, the Krone equation is commonly used to predict deposition in the range $\phi_l < \phi < \phi_h$, where $\phi_h \approx 0.002$–0.004 is the upper limit of ϕ above which the rate of fall of particles is characteristically hindered by upward seepage of water in the settling slurry; see

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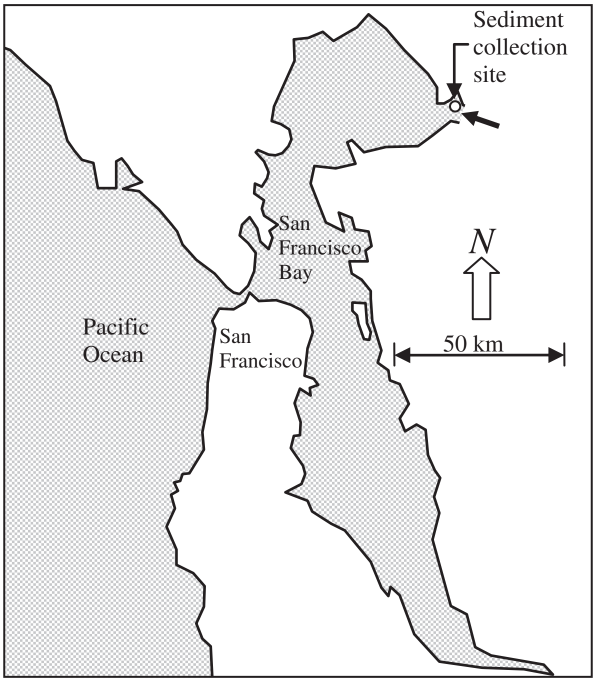

Fig. 1. The San Francisco Bay and Mare Island Strait sediment collection site. Thick arrow close to sediment collection site in studies of Krone (1962) and Mehta (1973) implies sediment inflow from river.

Winterwerp and van Kesteren (2004) for a discussion on the meaning of ϕ_h. We argue that in place of using the Krone equation as a purely empirical construct at high solids volume fractions, a more accurate expression for deposition must be employed. Such an expression, accounting the effects of ϕ and G on settling floc properties, is readily obtained by relying on Krone's own experiments with the Bay sediment as shown previously by Shrestha and Orlob (1996).

2. Flume experiments

Bay mud collected from Mare Island Strait in the northwestern part of the estuary (Fig. 1) was tested in a 30 m long and 0.9 m wide flume with a flow-return pipe and a pump to recirculate saltwater and sediment. The water depth h was kept at 0.3 m. After fully mixing sediment and water (at salinity $S = 17‰$ and nominal density $\rho_w = 1025$ kg m^{-3}) at a high flow velocity u_m, the velocity was reduced and held constant at 0.107 m s^{-1} (Table 1). Suspended sediment concentration C taken as the depth-mean value was measured using both an optical sensor and gravimetric analysis of samples of suspension withdrawn at specific times.

In addition to five deposition test runs at different flow velocities and initial concentrations, a separate deposition test was carried out in the same flume but at a slightly lower salinity ($S = 15‰$). In this test the significance of aggregation was qualitatively revealed by mechanically changing flow turbulence. To do so an industrial metallic grid (of unreported opening) was inserted across the flow area close to the flume head, thus producing a drastic increase in the rate of deposition downstream. More recent grid insertion experiments using sand have provided confirmatory data on the increase in turbulent intensity downstream of the grid (e.g. Sumer et al., 2003).

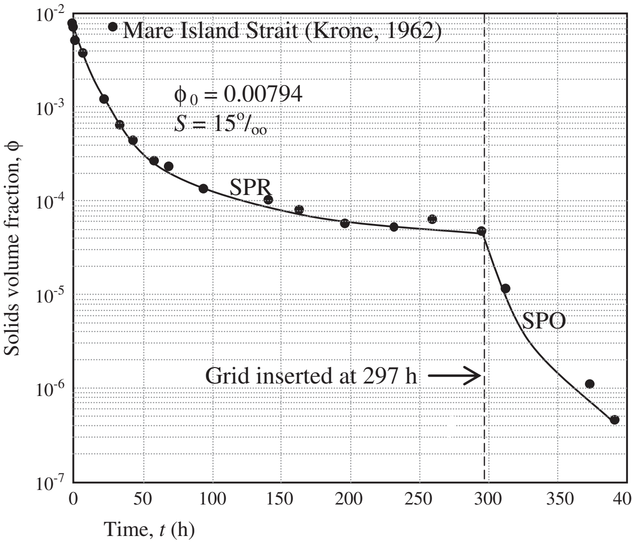

Fig. 2. Variation of solids volume fraction with time in grid insertion test of Krone (1962). Lines indicate mean trends.

In Fig. 2, the solids volume fraction ϕ is plotted against time (t) starting with the initial solids volume fraction $\phi_0 = 0.008$. At this high value settling was hindered until ϕ was below about 0.004 (Krone, 1962). Given mass settling flux $F_s = \rho_s w_s \phi$, in the first phase (SPR) of the test, ϕ decreased rapidly at first in spite of hindrance to settling, with $F_s = 0.72$ kg m^{-2} h^{-1} representing the initial rate of deposition. The flux then decreased gradually and reached a small value of about 0.0014 kg m^{-2} h^{-1} just before the grid was inserted at 297 h. This increased F_s in the second phase (SPO) due to aggregation by nearly an order of magnitude to 0.030 kg m^{-2} h^{-1}. The test was terminated at 392 h, when F_s had reduced to a negligible 0.0004 kg m^{-2} h^{-1}.

3. Deposition rate equation

The effect of aggregation on the settling flux F_s is deduced from particle balance. The rate of change of the instantaneous floc diameter $d_f(t)$ is equated to the difference between the rates of floc growth r_g and break up r_b with respect to the number concentration of particles (e.g. Overbeek, 1952):

$$\frac{dd_f}{dt} = r_g - r_b. \tag{1}$$

Inter-particle encounters leading to floc growth are promoted by diffusion associated with small fluid eddies and, as a result, for a floc of

Table 1
Sediment properties.

Test	Source	Fluid	ϕ_0 (−)	τ_d (Pa)
SR	Mare Island Strait, California	Saltwater ($S = 17‰$)	1.72×10^{-4}–3.28×10^{-3}	0.081
SC	Mare Island Strait, California	Native water + water with $S \approx 5‰$ NaCl	2.72×10^{-4}	0.065
MM	Maracaibo estuary, Venezuela	Native water + water with $S \approx 5‰$ NaCl	7.88×10^{-4}–1.03×10^{-3}	0.150
KD	Mined kaolinite, Florida	Distilled water	4.00×10^{-4}–3.64×10^{-3}	0.180
KS	Mined kaolinite, Florida	Saltwater ($S \approx 5‰$ NaCl)	2.32×10^{-3}–3.88×10^{-3}	0.150
SPR	Mare Island Strait, California	Saltwater ($S = 15‰$)	3.73×10^{-3}	0.081
SPO	Mare Island Strait, California	Saltwater ($S = 15‰$)	1.11×10^{-4}	0.081