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Geometry models of porous media based on Voronoi tessellations and their porosity–permeability relations

Feng Xiao, Xiaolong Yin *

Petroleum Engineering Department, Colorado School of Mines, Golden, CO 80401, USA

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ABSTRACT

In this paper, we present methods that directly model the random structure of porous media using Voronoi tessellations. Three basic structures were generated and they correspond to porous medium geometries with intersecting fractures (granular), interconnected tubes (tubular), and fibers (fibrous). Fluid flow through these models was solved by a massively parallelized lattice Boltzmann code. We established the porosity–permeability relations for these basic geometry models. It is found that, for granular and tubular geometries, the specific surface area is a critical structural parameter that can bring their porosity–permeability relations together under a unified Kozeny–Carman equation. A connected fracture network, superimposed on the basic Voronoi structure, increases the dimensionless permeability relative to the Kozeny–Carman equation; isolated large pores (vugs), on the other hand, decreases the dimensionless permeability relative to the Kozeny–Carman equation. The Kozeny–Carman equation, however, cannot distinguish a heterogeneous structure with an embedded partially penetrating fracture. The porosity–permeability relation for fibrous geometries in general agrees with those established for simple-cubic, body-centered cubic, and face-centered cubic models. In the dilute limit, however, the dependence on the solid fraction is weaker in Voronoi geometries, indicating weaker hydrodynamic interactions among randomly interconnected fibers than those in the idealized models.

1. Introduction

A porous medium is a type of two-phase disordered material, in which one phase is solid and the other phase is pore space, which must be continuous to allow flow and transport to take place. Artificial porous media include concrete, ceramics, paper, porous semiconductors and polymer composites; natural porous media include coral, bones, organic tissues, soil and rocks. Hydrocarbon-bearing rocks, in particular, are important resources, from which crude oil and natural gas are extracted.

In terms of geology, a hydrocarbon bearing rock can be sandstone, carbonate or shale. *Sandstone* is a clastic sedimentary rock made up mainly by sand-size (62.5 μm–2 mm) minerals that commonly are quartz and/or feldspar; the sand grains are cemented by crystallized silicates or calcium carbonates, the pore size is usually greater than 2 μm for conventional sandstone reservoirs and ranges from 0.03 to 2 μm for tight-gas sandstone reservoirs. *Carbonate* is formed by many physicochemical and biochemical processes (e.g. cementation, bioturbation, compaction and pressure solution, mineral replacement and recrystallization, and dolomitization) that act on terrestrial sediments mixed with organic debris such as shells or skeletons of mollusks and brachiopods; they may contain vuggy pores that are connected through interparticle

* Corresponding author.
E-mail address: xyin@mines.edu (X. Yin).

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porosity [1], and the pore size can vary from 10 nm to 10 cm [2]. *Shale* is a very fine-grained sedimentary rock that is formed by silt and clay-size (1–62.5 μm) minerals such as quartz and feldspar; it has a high percentage of kerogen if it is organic-rich and may also contain carbonate and pyrite. There are microfractures, pores in kerogen, and pores in minerals for gas shales, and the pore size can vary from a few nanometers to several hundred nanometers. These descriptions are brief for simplicity, and cannot cover all the geometric complexities. Of these rocks, the pore structure of shales and low permeability sandstones are still under active investigations for better characterization (e.g. [3–8]). The study by Nelson [9] provides a good summary of pore sizes in sandstones, tight sandstones and shales.

Flow and transport of hydrocarbon in these geological porous media are long-lasting research topics since the modern petroleum industry emerged in the 19th century. Central to this subject is the law discovered by Darcy [10] that treats the porous media as a continuum and relates flow velocity to pressure differential by the use of permeability or hydraulic conductivity. Today we know that the Darcy's law results from the upscaling of slow viscous laminar flow of an incompressible Newtonian fluid. The practical importance of Darcy's law stimulated researchers to derive the permeability from the knowledge of the microstructure of porous media. Although the simplest way to characterize pore structures is the void fraction, namely porosity, there have been many other approaches that have gone to greater details. Some are purely empirical expressions that relate permeability to an effective grain diameter (e.g. [11–13]) or to packing- and sand-shape factors (e.g. [14]); others are based on more quantitative models of the pore structure that may be further divided into two classes: the *cell model* in which the fluid phase is continuous and solid phase is discontinuous ([15,16]), and the *bundle or network of capillary tubes* in which the solid phase is continuous with interconnected fluid-filled pores. The well-known Kozeny–Carman equation [17–19] was developed based on Poiseuille flow through a bundle of capillary tubes, and it yields permeability as a function of porosity, specific surface area and a coefficient that takes into account the shape and tortuosity of tubes. The widely used network model treats the porous medium as a network of pore bodies of varying sizes and pore throats of varying cross sections that connect the pores. The first network model was composed of regular networks of single-size or varying-size tubes proposed by Fatt [20–22]. In addition to these two approaches, there are also attempts made to extract statistical information, such as statistical correlation functions, of the pore structure for permeability estimation [23]. However, it is only possible to derive bounds on the permeability due to the limited information from this statistical approach. This method, therefore, is known under the name of *variational bounds*. de Boer et al. [24] provides a historical review including continuum theory and void fraction theory. The reference by Adler [25] includes more details and references for these pioneering studies.

Besides the above mentioned intuitive methods, researchers have also applied other theories to study porous media. The first is *percolation theory* introduced by Broadbent and Hammersley [26], and it deals with completely random materials without any spatial correlation. The early studies of percolating structures were carried out on random square or cubic lattice structures (e.g. [27]); later, 3D models were generated using continuum objects placed at random or regular positions (e.g. [28]). For simulating fluid flow in porous media, networks of tubes [29,30] are mostly used, and usually studied is the relation between the permeability and the critical percolation threshold (e.g. [31,32]). Golden [33] provides a complete review of percolation models for porous media, and an in-depth introduction to percolation theory and its applications for flow in porous media is available in Hunt and Ewing [34]. The second is *fractals* invented by Mandelbrot [35] for geometries that can be characterized by statistical self-similarity. Fractal dimension has been used to quantify surface roughness and its morphology for sandstones (e.g. [36]), carbonates (e.g. [37]) and fractures [38]. Fractal geometries can also be directly applied to simulate porous media, for example, the Navier–Stokes equation in conduits (pores) with fractal perimeter was solved for permeability (e.g. [39,40]). The references by Adler [25] and Sahimi and Yortsos [41] have good reviews on the application of fractals to porous media.

Numerous investigations have also been conducted to reproduce the real microstructure of porous media, and they are named after *reconstruction*. Several reconstruction techniques have been developed owning to the advances made in spatial microscopy technology, numerical simulations aided by three-dimensional visualization and applications of methods from disciplines such as statistical physics, spatial analysis and digital image analysis. The first reconstruction technique is direct measurement of a 3D microstructure via X-ray computed tomography (CT) (e.g. [42,43]), focused ion beam-scanning electron microscope (FIB-SEM) with submicron resolution (e.g. [44,3,6]), or energy-dispersive spectrometry (EDS) that is also capable of giving mineralogical compositions (e.g. [45,46]). Wildenschild and Sheppard [47] provide a thorough and latest review of these imaging techniques. The second reconstruction technique is to generate 3D microstructures from spatial information measured on 2D images (e.g. [48]). This topic has been extensively studied largely due to the convenient and inexpensive access to high-resolution 2D images such as thin sections from simple optical techniques or cross section images from other advanced imaging techniques. The process consists of two steps: one step is to measure structural information on 2D images such as the commonly used porosity and statistical correlation functions; the other step is to numerically generate 3D images to match those measures using threshold Gaussian random fields (e.g. [49,50]), Boolean or germ-grain model (e.g. [51]) and simulated annealing (e.g. [52–56]), and recently introduced Markov random field based method [57]. However, these reconstructed models from 2D images are still not the geometrical (pore size and shape) and topological (pore connectivity) equivalents of the original sample [58,59], although they are equivalent in terms of the applied statistical measures, they may generate poor connectivity when dealing with low porosity. Along the line of imaging and statistical measures, recently there is an interesting work that a $0.4 \times 0.4 \times 17$ m^3 of karst carbonate was reconstructed with a resolution of 5 mm from borehole imagery and caliper data [60]. The third reconstruction technique is to model the geological process by which the porous medium was made [61–63]. By simulating the processes of sedimentation, compaction and cementation