Computers and Mathematics with Applications 72 (2016) 394–403

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

A 9-bit multiple relaxation Lattice Boltzmann magnetohydrodynamic algorithm for 2D turbulence

Christopher Flint [a], George Vahala [a,*], Linda Vahala [b], Min Soe [c]

[a] Department of Physics, William & Mary, Williamsburg, VA 23187, United States
[b] Department of Electrical & Computer Engineering, Old Dominion University, Norfolk, VA 23529, United States
[c] Department of Mathematics and Physical Sciences, Rogers State University, Claremore, OK 74017, United States

ARTICLE INFO

Article history:
Available online 9 October 2015

Keywords:
Lattice Boltzmann
MHD
2D turbulence
Single and multiple relaxation

ABSTRACT

While a minimalist representation of 2D Magnetohydrodynamics (MHD) on a square lattice is a 9-bit scalar and 5-bit vector distribution functions, here we examine the effect of using the 9-bit vector distribution function on the effect of a magnetic field on the Kelvin–Helmholtz instability. While there is little difference in the simulation results between the 5-bit and the 9-bit vector distribution models in the vorticity, energy spectra, etc., the 9-bit model permits simulations with mean magnetic field a factor of approximately $\sqrt{2}$ greater than those attainable in the standard 5-bit model. Indeed a 9-bit single-relaxation model can attain such success over a 5-bit multiple-relaxation model at the same computational expense.

1. Introduction

Lattice Boltzmann (LB) algorithms [1] are an extremely successful computational technique for solving nonlinear collisional problems because of their simplicity of coding and their ideal parallelization. In principle, one is replacing a computationally difficult problem involving nonlinear convective terms by a linearized kinetic equation with simple advection and local collision operator. Since one is now solving on a lattice in kinetic space, one typically reduces the inherited extra memory/calculations by minimizing the number of kinetic velocities required in LB to recover the fluid equations in the long wavelength limit. In particular, such a minimalist 2D LB MHD model has been introduced by Dellar [2]. It consists of a 9-bit square lattice model for the evolution of the quasi-incompressible fluid velocity **u** and a 5-bit (square lattice) model for the evolution of the magnetic field **B**. This asymmetry is because the fluid velocity **u** arises as the 1st moment but the magnetic field arises from the 0th moment of their corresponding distributions. Here we consider in some detail the effect of utilizing the same 9-bit square lattice for the vector magnetic distribution function as for the particle distribution function on the magnetic stabilization of the Kelvin–Helmholtz instability [3,4]. In considering the stabilization of a velocity jet we extend the collision operator for the evolution of the vector magnetic distribution to a multiple relaxation model (MRT) — a relaxation model introduced in 2002 by d'Humieres et al. [5] for LB for Navier–Stokes flows and which permitted numerically stable LB simulations for considerably higher flow velocities than could be achieved by the single relaxation rate (SRT). Similarly, we [6] have found that an MRT extension to the evolution of the 5-bit vector distribution function permitted considerably greater **B**-fields than under the SRT collision operator. (We note that MRT models for LB MHD have also been considered earlier by Dellar [7,8], but these papers were not concerned with the question we are

* Corresponding author.
E-mail address: gvahala@gmail.com (G. Vahala).

http://dx.doi.org/10.1016/j.camwa.2015.09.008

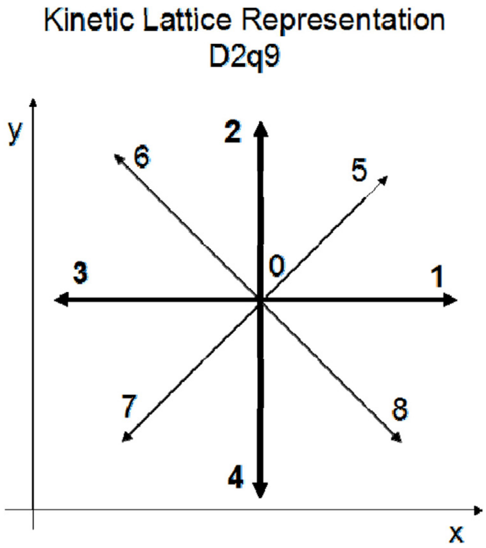

Fig. 1. The D2Q9 velocity lattice for our 2D LB-MHD simulations.

investigating.) Here we will find that even a single relaxation rate (SRT) model of the 9-bit collision operator leads to stable simulations with magnetic field magnitudes on the order of 34% greater than those attainable from the 5-bit MRT simulations at fixed Reynolds numbers. Moreover, this can be achieved at the same computational expense (wallclock time). Moving to 9-bit MRT model only gains a further 6% in maximum B-field over the 9-bit SRT, but with a wallclock time increase of over 50%. These results have direct bearing on LB-MHD tokamak simulations with its much larger toroidal to poloidal magnetic fields.

2. Lattice Boltzmann algorithms

For 2D LB MHD [2], the minimalist square lattice is 9 streaming velocities for the evolution of the scalar density distribution function, Fig. 1, but 5 streaming velocities for the vector magnetic distribution $[\{\mathbf{e}_i,\ i = 0\ldots,4\}]$. However, here we will consider using exactly the same kinetic lattice velocities for both distributions — the D2Q9 model.

The lattice vectors are along the axes and the diagonals of a unit square

$$\mathbf{e}_i = (0,0)\,,\ (\pm 1,0)\,,\ (0,\pm 1)\,,\ \ (\pm 1,\pm 1)\,,\quad i = 0\ldots 8. \tag{1}$$

The 2-step SRT lattice algorithm (with relaxation rate) for the scalar distribution $f_i\,(\mathbf{x},t)$ consists of astream–collide sequence

$$\begin{aligned}&\text{Stream: } f_i'\,(\mathbf{x},t+\delta t) = f_i\,(\mathbf{x}-\mathbf{e}_i\,\delta t,\ t)\\ &\text{Collide: } f_i\,(\mathbf{x},t+\delta t) = f_i'\,(\mathbf{x},t+\delta t) + \frac{1}{\tau}\left[f_i^{eq}\,(\rho,\mathbf{u},\mathbf{B}) - f_j'\,(\mathbf{x},t+\delta t)\right]\end{aligned} \tag{2}$$

where the streaming is a shift of the data from one spatial lattice node to a neighboring node, while where the streaming is a shift of the data from one spatial lattice node to a neighboring node, while the collision step requires only local on-site information. Similarly, for the SRT evolution of the vector magnetic distribution $\mathbf{g}_i\,(\mathbf{x},t)$

$$\begin{aligned}&\text{Stream: } \mathbf{g}_i'\,(\mathbf{x},t+\delta t) = \mathbf{g}_i\,(\mathbf{x}-\mathbf{e}_i\delta t,\ t)\\ &\text{Collide: } \mathbf{g}_i\,(\mathbf{x},t+\delta t) = \mathbf{g}_i'\,(\mathbf{x},t+\delta t) + \frac{1}{\tau_m}\left[\mathbf{g}_i^{eq}\,(\rho,\mathbf{u},\mathbf{B}) - \mathbf{g}_i'\,(\mathbf{x},t+\delta t)\right],\quad i = 0\ldots 8.\end{aligned} \tag{3}$$

From Chapman–Enskog theory, the kinetic Eqs. (2) and (3) are coupled through the **u**- and **B**-fields in the relaxated distributions, f^{eq} and $\mathbf{g}^{eq}$ [2]

$$\begin{aligned}f_i^{eq} &= w_i\rho\left[1 + 3\,(\mathbf{e}_i\cdot\mathbf{u}) + \frac{9}{2}\,(\mathbf{e}_i\cdot\mathbf{u})^2 - \frac{3}{2}\mathbf{u}^2\right] + \frac{9}{2}w_i\left[\frac{1}{2}\mathbf{B}^2\,\mathbf{e}_i^2 - (\mathbf{B}\cdot\mathbf{e}_i)^2\right] + O\left(u^4\right)\\ \mathbf{g}_i^{eq} &= w_i\left[\mathbf{B} + 3\left\{(\mathbf{e}_i\cdot\mathbf{u})\ \mathbf{B} - (\mathbf{e}_i\cdot\mathbf{B})\ \mathbf{u}\right\}\right] + O\left(u^4\right)\end{aligned} \tag{4}$$

with the same weights $w_o = 4/9,\ w_{1\ldots 4} = 1/9,\ w_{5..8} = 1/16$.