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Computational homogenisation of composite plates: Consideration of the thickness change with a modified projection strategy

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A B S T R A C T

In the present paper, a method for the modelling of the mechanical behaviour of composite plates, especially hybrid laminates and sandwich plates, is proposed. The chosen method is the computational homogenisation, or the so-called FE^2, for plates. The principle is to split the considered problem into two scales: on the one hand, the two-dimensional FE computation of a plate is performed on the macroscale. On the other hand, a three-dimensional FE problem is computed on the mesoscale, which enables the discretisation of the layers organisation. From each integration point of the macroscale, the deformations are projected to the Representative Volume Element (RVE) on the mesoscale, where a Dirichlet boundary value problem is solved. Finally, the homogenisation of the stresses is conducted to define the stress resultants of the macroscale. The proposed method presents the advantage to take into account any material behaviour without any transformation of the constitutive laws, even non-linear ones. Furthermore, the new projection strategy, presented in this work, considers the thickness change of the plate, which enables a resolution of the Poisson locking. The method is illustrated by the simulation of the mechanical behaviour of both sandwich plate and of hybrid laminate.

1. Introduction

Nowadays composite plates, such as sandwich plates and hybrid laminates, are widely used. The composite plates find application in the transport industry, because of their outstanding mechanical properties at a relatively low weight. However, the mechanical behaviour of composite plates is complex and it can be expensive to test them experimentally. Therefore, the modelling of this kind of structures, which also enables the optimisation of the layers organisation, is of interest. In the present paper, two types of composite plates are considered: on the one hand, a three-layers sandwich plate, and on the other hand, a hybrid laminate. The hybrid laminate is composed of the superposition of different layers, namely $(\text{metal}/\text{CFRP}(0/90/0)/\text{metal})_S$, with a symmetric layers organisation. The metal layers present an elasto-plastic material behaviour and the CFRP (Carbon Fibres Reinforced Polymer) layers show a transverse isotropic elastic material behaviour. The sandwich structure is composed of elasto-plastic top panels and of an isotropic elastic core. Consequently, linear and non-linear material behaviours have to be considered in this work, which excludes most of the classical plate theories derived from the continuum theory, following the appellation given by Naghdi [1].

A review of the plate theories derived from the continuum theory can be found in [2–4], among others. The first correct attempt to propose a plate theory can be attributed to Kirchhoff, cf. [5], and is focused on thin plates. The plate theory following the Kirchhoff ansatz [6] studies the case of a bending plate, i.e. a plane stress problem where only the transverse

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displacements are considered and the in-plane displacements ignored. Combined with a membrane problem, it can be obtained that the displacements in the three directions are described by three degrees of freedom. The two rotations of the cross-section are defined as the derivative of the transverse displacement with respect to the two in-plane coordinates. It is assumed that the cross section, straight and normal to the plate's midplane before deformation, stays straight and normal to the plate's midplane after deformation. The out-of-plane shear strains are not considered, which leads to accurate results for plates whose ratio of thickness to length is smaller than $1/10$. Later on, some improvements proposed independently by Reissner [6] and Mindlin [7] lead to the treatment of thicker plates, with the ratio of thickness to length going to $1/10$, cf. [8]. Two more independent degrees of freedom are introduced, which account for the rotations. Following this ansatz, the cross section, straight and normal to the plate's midplane before deformation, stays straight but can afford a rotation after deformation. However, most of the plate theories following the Kirchhoff or the Mindlin ansatz are restricted to linear material behaviour, to the knowledge of the authors, cf. [4,8]. Furthermore, the constitutive law is two-dimensional, the thickness change of the plate is not taken into account, and the finite element formulations suffer many locking problems, cf. [9–12].

An improvement considering the thickness change was proposed by Krätzig [13], although the consideration of the thickness change could have been proposed previously by Hildebrand, Reissner and Thomas in 1949, cf. [14], according to Bischoff [9] and Carrera [4]. Considering the plate theory with thickness change, there are actually two different concepts which enable a thickness change. In the first concept, two extra degrees of freedom are added to the transverse displacements to obtain a quadratic displacement in thickness direction, cf. [9,15–17]. The theory is called the seven-parameters theory or the $(1, 1, 2)$-plate model, in reference to the order of the displacement in the three directions. The second concept is the EAS method, cf. [9,18–20], where an extra degree of freedom is introduced locally, in order to enable also a linear distribution of the normal deformation in thickness direction. For both concepts, the thickness change is considered and this introduces a solution of the Poisson locking. In the framework of this paper, the Poisson or volume locking is defined according to [9], as the occurrence of parasitic normal stresses in thickness direction. This locking effect does not exist for a material with a Poisson ratio equal to zero, and becomes more important for an incompressible material behaviour with the Poisson ratio tending to $\nu = 0.5$. According to [9], the Poisson locking does not disappear with a finer mesh, because the refinement does not occur in the thickness direction for plates. For the two concepts, the constitutive law is then a three-dimensional one, and it is possible to consider non-linear material behaviour. However, these solutions have drawbacks, as for instance if considering an elasto-plastic material law, that the "additive form does not satisfy patch incremental test", cf. Roehl et al. [20]. Moreover, a plate theory is considered as consistent if the order of the displacement in the thickness direction is smaller than the order of the displacement in the two in-plane directions, cf. [21]. Because of these drawbacks, the presented paper aims at the modelling of the mechanical behaviour of composite plates using a numerical concurrent homogenisation method.

Several multi-scale methods exist, in order to describe the behaviour of composite materials, as referred by Kanouté et al. [22]. There is the possibility to use the analytical homogenisation for plates, cf. Laschet et al. [23]. Another possibility is the numerical homogenisation, where separate FE computations for the macroscale and the mesoscale are done, as proposed by many authors, among them Hohe [24] or Fish and Wagiman [25]. The specific case of masonry was studied thereafter, cf. [26]. A numerical homogenisation performed with the commercial software ABAQUS® was developed by Oskay and Pal in [27] for thin heterogeneous plates including damage. The modelling of failure in adhesive layers was done within the framework of cohesive zones, cf. [28], and the simulation of the behaviour of cohesive zones for multiscale plates was proposed by [29]. The numerical homogenisation was also performed using the Bending-Gradient theory, cf. [30,31], whose principle consists in the separation of the constituents of the gradient of the bending moments, enabling a variation of the constitutive material through the thickness.

Another possibility to model the behaviour of composite plates is to use a numerical concurrent homogenisation, so-called FE^2 or computational homogenisation, for plates. It is based on the linking of the FE computations of the macroscale and the mesoscale. It was developed by Geers et al. for plates, cf. [32], or by Grytz and Meschke for shells, cf. [33]. Coenen et al. proposed a computational homogenisation based on a Kirchhoff or a Reissner–Mindlin type plate, cf. [34,35]. The FE^2 method for plates based on a Mindlin plate theory presents the problem of the Poisson locking, cf. [36], because the thickness change is not considered in the macroscale for the plate theory following the Mindlin ansatz. Therefore, an improvement considering a plate theory with thickness change can be proposed, cf. [37]. Landervik and Larsson present the modelling of porous layers using a first and second order multi-scale homogenisation for shells with a thickness change, cf. [38,39]. However, this solution encounters convergence problems, which can be related to the consistency of the plate theory with seven degrees of freedom. Therefore, a modified projection strategy is proposed in this work.

The principle of the FE^2 method for plates is represented in Fig. 1 and consists of four steps, cf. [35,40]. In a first step, accurate plate kinematics are defined. In the framework of the FE^2 method, the plate theory following the Mindlin ansatz is chosen. But instead of using the constitutive law for the plates, the deformations are projected from each integration point of the macroscale to the three-dimensional mesoscale. In the mesoscale, the Representative Volume Element (RVE) is defined, which has to be large enough to represent the heterogeneities of the mesoscale, but not too large, in order to avoid a slow down of the computations. In the RVE, a Dirichlet boundary value problem is solved, using the equilibrium equation and the constitutive equations for the different materials. It has to be mentioned that an important advantage of the FE^2 method is that any constitutive law can be considered, and that no further modification of the constitutive law is needed. Finally, the stress resultants of the macroscale are defined using the Hill–Mandel condition.