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He's frequency–amplitude formulation for the Duffing harmonic oscillator

Jie Fan

TMT Laboratory, College of Textiles, Donghua University, Shanghai 201620, PR China

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ABSTRACT

He's frequency–amplitude formulation is used to solve the Duffing harmonic oscillator problem. The solution procedure is simple, and the result obtained is valid for the whole solution domain with high accuracy.

1. Introduction

The study of nonlinear periodic oscillators is of interest to many researchers. Several approaches have been proposed for dealing with the Duffing harmonic oscillator, for example, the variational iteration method [1–5], He's homotopy perturbation method [6–11], the parameter-expansion method [12–14], and He's frequency formulation [15,16].

He's frequency–amplitude formulation [17] suggested by J. H. He is a simple and effective method for solving nonlinear oscillatory problems; the formulation was further improved by the originator [18,19]. In this work, He's frequency–amplitude formulation is employed to solve the Duffing harmonic nonlinear oscillator problem.

2. Solution procedure

We first consider the following nonlinear oscillator:

$$u'' + u^3 + \frac{u}{1+u^2} = 0, \quad u(0) = A, \ \frac{\mathrm{d}u}{\mathrm{d}t}(0) = 0. \tag{1a}$$

We rewrite Eq. (1a) in the form

$$(1+u^2)(u'' + u^3) + u = 0, \quad u(0) = A, \ \frac{\mathrm{d}u}{\mathrm{d}t}(0) = 0. \tag{1b}$$

Equations $u_1(t) = A\cos t$ and $u_2(t) = A\cos\omega t$ serve as the trial functions. Substituting the above trial functions into (1b) results in, respectively, the following residuals:

$$R_1(t_1) = A^5 \cos^5(t_1) \tag{2}$$

and

$$R_2(t_2) = \left(1-\omega^2\right) A\cos\omega t_2 + \left(1-\omega^2\right) A^3 \cos^3\omega t_2 + A^5 \cos^5 \omega t_2. \tag{3}$$

E-mail address: fanjie@mail.dhu.edu.cn.

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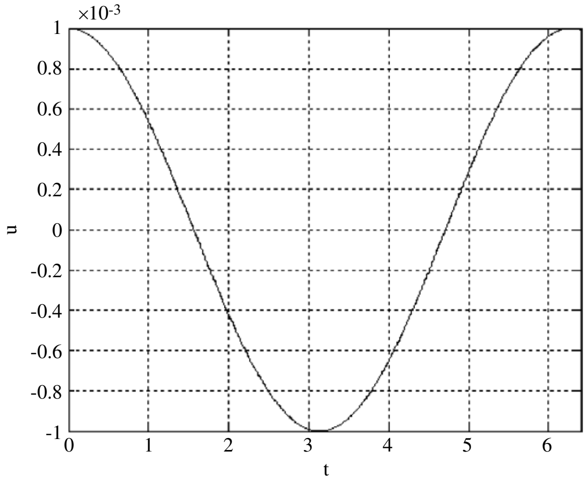
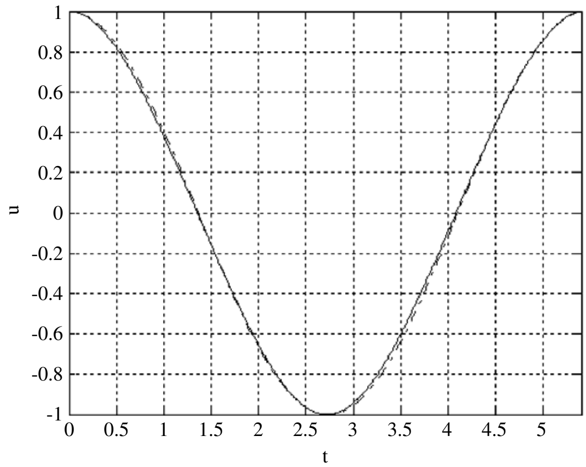
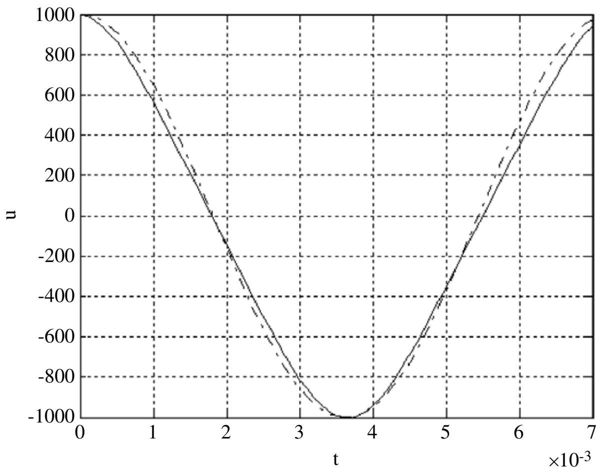


(a) A = 0.001.



(b) A = 1.



(c) A = 1000.

Fig. 1. Comparison of the approximate solution, $u = A\cos\omega t$, with the numerical solution. Continuous line: exact solution, dashed line: approximate one.

The frequency–amplitude formulation reads [18,19]

$$\omega^2 = \frac{\omega_1^2 R_2(t_2) - \omega_2^2 R_1(t_1)}{R_2(t_2) - R_1(t_1)} \tag{4}$$

where t_1 and t_2 are location points. The frequencies of u_1 and u_2 are respectively $\omega_1 = 1$ and $\omega_2 = \omega$; ω is the frequency of the nonlinear oscillator.

Generally we locate at the points

$$t_1 = \frac{T_1}{N}, \qquad t_2 = \frac{T_2}{N}$$

where T_1 and T_2 are periods of the trial functions $u_1(t) = A\cos t$ and $u_2(t) = A\cos\omega t$, respectively. Setting $N = 12$, we obtain

$$\omega^2 = \frac{\omega_1^2 R_2(t_2) - \omega_2^2 R_1(t_1)}{R_2(t_2) - R_1(t_1)} = \frac{3}{4}A^2 + \frac{1}{1 + \frac{3}{4}A^2}, \tag{5}$$

i.e.

$$\omega = \sqrt{\frac{3}{4}A^2 + \frac{1}{1 + \frac{3}{4}A^2}}. \tag{6}$$