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Extending a potential vorticity transport eddy closure to include a spatially-varying coefficient

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ABSTRACT

The use of spatially varying eddy diffusivities is explored with the extended Gent–McWilliams (eGM) closure for both passive tracers and potential vorticity (PV). Numerical experiments are conducted with a wind-forced isopycnal channel model. It is shown that, the eGM closure with eddy diffusivities derived from a high-resolution reference solution produces the best results compared to the reference solution in terms of the thickness, PV profiles and volume fluxes. The use of spatially varying eddy diffusivities also removes the unphysical reverse jets near the channel walls shown by the eGM with constant eddy diffusivities.

1. Introduction

The Gent–McWilliams (GM, Gent and McWilliams [1]; Gent et al. [2]; Gent [3]) closure is an essential component of non-eddy-resolving global ocean models used to study climate. GM serves as a surrogate for the baroclinic instability that is largely responsible for the existence of mesoscale eddies which, in turn, transport a substantial amount of heat polewards. The closure is based on the fundamental assumption that the eddy transport should be down the thickness gradient, and along, not across, isopycnal surfaces. GM is used in conjunction with mixing along isopycnal surfaces that was formalized by Redi [4]. While the closure has seen widespread success in ocean modeling since its introduction (see e.g. [5,6] among many others), there are also aspects of the closure that suggest improvements are possible. For example, in many implementations of GM (exceptions will be discussed below), the closure depends on two parameters, the thickness diffusivity and the isopycnal diffusivity, which are frequently taken as constants in the horizontal and vertical directions. It is natural to ask whether spatially varying parameters are more appropriate.

Many efforts have been made to determine and implement spatially-varying GM parameters. One of the first was Visbeck et al. [7] who proposed that the closure parameter is proportional to the square of the width of the baroclinic zone divided by the time scale determined from the Coriolis parameter and the local Richardson number. There are too many proposals to provide a complete list here. However, very recently Farneti and Gent [8] use the GFDL CM2.1 climate model to test a spatially varying formulation of the eddy-induced advection parameter that is proportional to the vertical average of the horizontal gradients of the potential density field. Hofmann and Morales Maqueda [9] test a formulation close to that of Visbeck et al. [7] in a global ocean model. Both Farneti and Gent [8] and Hofmann and Morales Maqueda [9] confirm that a

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spatially varying formulation of the eddy diffusivity parameter helps to reproduce the insensitivity of the Southern Ocean circulation to strongly intensified Southern Hemisphere westerlies seen in eddy-resolving models.

There is also a long history suggesting the parametrization of mesoscale eddy transport should be based on mixing of the potential vorticity (PV) field, due to its tracer like property; e.g. [10–15]. Ringler and Gent [16] extend the standard GM closure to the momentum equations utilizing the similarity between the PV equation and the equation for a tracer. Energetic analysis shows that, in an adiabatic system with the extended GM closure, the total energy is conserved up to a small error proportional to the temporal changing rate of the bolus velocity (see Eq. (17) therein). The last point is essential because it implies that the extended GM closure preserves the conversion between the available potential energy (APE) and the kinetic energy (KE), up to a small error term. It is well known that the standard GM closure provides a monotone APE sink. The extended GM closure is evaluated in a three-layer isopycnal model for the Antarctic Circumpolar Current (ACC). The standard GM (labeled GMST therein), the extended GM (labeled GMPV therein), together with the standard GM closure with the eddy induced PV transport only (labeled PVBL therein) are applied in the 62.5 km low resolution simulations, and the results are compared to the 10 km reference solution. The performance of the various eddy closures is evaluated in terms of the time mean and zonal mean of the zonal jet velocity and of the PV. The authors come to the conclusion that the extended GM configured with a small eddy mixing coefficient (250 m^2/s) is marginally better than the other configurations. However, the extended GM simulations show strong westward currents along the southern and northern boundaries that do not appear in the high-resolution simulation. The authors conjecture that using spatially varying eddy diffusivities that, in particular, vanish near the boundaries can remedy this deficiency for the closure. A much broader question, which is implicit but left un-explored in [16], is whether using spatially varying eddy diffusivity has any merit in simulating the world ocean.

The goals of this work are two-fold. The first is to test the hypothesis made in [16], namely that using spatially varying eddy diffusivities can remedy the deficiency of the extended GM closure mentioned above. For this study, we derive the spatial distribution of the eddy diffusivity using a high-resolution, eddy resolving simulation. Second, in a general setting, we want to answer the question whether using spatially varying eddy diffusivities can improve overall the results of model simulations that do not permit mesoscale eddies. We approach these questions with a set of controlled simulations using a three-layer isopycnal model for the ACC. A 10 km simulation of the model will provide the reference solution for this study. The goal of any subgrid parametrization is to replicate certain features of a high resolution simulation in a low resolution setting. For the latter, we use a 125 km mesh so that the mesoscale eddies are not resolved. In the current study, we do not develop or test any premises on the formulation of the eddy diffusivity. Instead, we use the high resolution data to derive the point-wise eddy diffusivities for each layer, and then feed the derived parameters into the low resolution simulations. We note that a similar approach has been taken in [15], also in a channel domain, although his model is for zonally-averaged quantities and uses a different forcing technique than wind forcing. Care has to be taken with this approach because the parameters derived from the high resolution simulation may have deficiencies (e.g. negative diffusivities) that render the low resolution simulations oscillatory or even unstable.

2. The extended Gent–McWilliams (eGM) closure

A three-layer isopycnal system

For this study, we employ an adiabatic three-layer isopycnal model described by three sets of equations, one for each layer,

$$\begin{cases} \dfrac{\partial h_i}{\partial t} + \nabla\cdot(h_i\mathbf{u}_i) = 0, \\ \dfrac{\partial \mathbf{u}_i}{\partial t} + h_i q_i \mathbf{k}\times\mathbf{u}_i = -\nabla\left(\dfrac{\phi_i}{\rho_0} + K_i\right) + \mathbf{D}_i + \mathbf{F}_i, \\ \dfrac{\partial}{\partial t}(h_i\sigma_i) + \nabla\cdot(h_i\sigma_i\mathbf{u}_i) = 0, \end{cases} \tag{1}$$

where $i = 1, 2, 3$ is the layer index starting at the ocean surface. The prognostic variables h_i, $\mathbf{u}_i$ and σ_i denote the layer thickness, horizontal velocity, and some tracer respectively, and the diagnostic variables q_i, ϕ_i and K_i denote the potential vorticity, Montgomery potential and kinetic energy, respectively, and they are defined as

$$\begin{aligned} q_i &= \frac{\nabla\times\mathbf{u}_i + f}{h_i}, \quad i = 1, 2, 3, \\ K_i &= \frac{1}{2}|\mathbf{u}_i|^2, \quad i = 1, 2, 3, \\ \phi_1 &= p_0 + \rho_1 g(h_1 + h_2 + h_3 + b), \\ \phi_2 &= \phi_1 + (\rho_2 - \rho_1)g(h_2 + h_3 + b), \\ \phi_3 &= \phi_2 + (\rho_3 - \rho_2)g(h_3 + b), \end{aligned}$$