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Convergence studies in meshfree peridynamic simulations

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ABSTRACT

Meshfree methods are commonly applied to discretize peridynamic models, particularly in numerical simulations of engineering problems. Such methods discretize peridynamic bodies using a set of nodes with characteristic volume, leading to particle-based descriptions of systems. In this paper, we perform convergence studies of static peridynamic problems. We show that commonly used meshfree methods in peridynamics suffer from accuracy and convergence issues, due to a rough approximation of the contribution of nodes near the boundary of the neighborhood of a given node to numerical integrations. We propose two methods to improve meshfree peridynamic simulations. The first method uses accurate computations of volumes of intersections between neighbor cells and the neighborhood of a given node, referred to as partial volumes. The second method employs smooth influence functions with a finite support within peridynamic kernels. Numerical results demonstrate great improvements in accuracy and convergence of peridynamic numerical solutions when using the proposed methods.

1. Introduction

Material failure and damage simulation is an ongoing area of research in computational science and engineering. Modeling systems with evolving discontinuities represents a challenge in the classical theory of continuum solid mechanics, due to its differentiability assumption on displacement fields. To resolve this essential limitation, a nonlocal continuum theory called *peridynamics*, based on long-range interactions, was proposed in [1,2]. Constitutive models in peridynamics depend on finite deformation vectors, as opposed to classical constitutive models which depend on deformation gradients. As a consequence, discontinuities in displacement fields can be naturally represented in peridynamics, making this theory suitable for the description of cracks and their evolution in materials. Applications of peridynamics include failure and damage in composite laminates [3–7], crack propagation and branching [8–10], crack nucleation [11,12], phase transformations in solids [13], impact damage [14–17], polycrystal fracture [18,19], crystal plasticity [20], structural health monitoring [21], damage in concrete [22], geomaterial fragmentation [23], and dynamic blast loading [24], among others.

Peridynamics is based on integro-differential equations, where spatial integration is employed to compute the contribution of internal forces in a body to the material response. Since governing equations in peridynamics are continuum models, they can be discretized in many ways [25]. Different discretization methods differ in software complexity, computational expense and memory requirements, and accuracy and convergence of numerical solutions. A simple

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particle-based discretization for the strong form of peridynamic equations was introduced in [15], where a set of nodes with known volume in a reference configuration was utilized to discretize given domains. This discretization method is meshfree, because no elements or geometrical connections between nodes are used. This meshfree approach is the most widely used discretization method in engineering peridynamic simulations to date, due to its implementation simplicity and relatively low computational cost, in comparison to other discretization methods. As an example, finite element discretizations of governing equations are based on weak forms, which for peridynamic equations double the number of spatial dimensions that need to be discretized [26]. In peridynamics, each material point is assumed to directly interact with a surrounding neighborhood, and the interaction is computed through spatial integration. In meshfree discretizations, integrals in peridynamic equations are converted into weighted sums. In [15], summation weights are taken as nodal volumes.

The accuracy and convergence of the above-mentioned meshfree discretization depends on the choice of summation weights. In fact, those weights can be interpreted as the volumes of the intersections between the neighborhood of a given node and the material regions or *cells* defining the nodal volumes of surrounding nodes [27]. For surrounding nodes near the boundary of the neighborhood of a given node, only a partial overlapping may exist between their cells and that neighborhood. In those cases, we refer to the volume of the corresponding intersection as a *partial volume*. Computing partial volumes requires, in general, complex geometrical calculations. For instance, in three-dimensional problems, partial volume calculations in meshfree discretizations of peridynamic equations with a set of nodes along a cubic grid require the computation of arbitrary intersections between a ball and a cube. Algorithms for approximations of partial volumes appear in [28–31]. In two or one dimensions, corresponding "partial volumes" are referred to as *partial areas* or *partial lengths*, respectively. In [27], partial areas were calculated analytically for sets of nodes along a square grid, resulting in improved accuracy and convergence of numerical integrations, for different peridynamic quantities of interest. In the remainder of this paper, unless specified otherwise, we will use the term *partial volume* in a general sense to refer to a partial volume, partial area, or partial length, in 3D, 2D, or 1D, respectively.

An alternative way to improve numerical integrations in peridynamics is to employ kernels which decay to zero at the boundary of the neighborhood of a given node. The idea behind this method is to reduce the contribution of neighboring nodes near the boundary of the neighborhood of a given node to the numerical integration, mitigating the discretization error induced by the inaccuracy of the approximation of partial volumes. This idea was briefly mentioned in [31] and implemented in [27], employing smooth influence functions with a finite support. Numerical studies in [27] suggested that this method could provide a means to improve the accuracy and convergence of numerical integrations in peridynamics.

In this paper, we present convergence studies of numerical solutions of static peridynamic problems using meshfree discretizations. In nonlocal models, the concept of convergence can be understood in more than one sense [32,33]. As in classical (local) problems, based on partial differential equations, one can study the convergence of numerical solutions to analytical ones under grid refinement. However, one can also consider the convergence of either a nonlocal model or the solution of a nonlocal problem to the corresponding classical one, in the limit of vanishing nonlocality. It is in the first sense that we perform convergence studies in this paper, i.e., convergence of numerical solutions of nonlocal problems to the analytical solutions of those problems under grid refinement, keeping the nonlocal length scale fixed. Using the method of manufactured solutions, we obtain analytical solutions to the problems studied here. Related convergence studies for nonlocal elliptic boundary value problems appear in [34]. The authors of that paper reported oscillatory convergence results and suggested to account for accurate calculations of partial volumes to achieve improved accuracy and convergence of numerical solutions, which is a central focus of this paper. We compute analytically partial lengths in 1D and partial areas in 2D, following [27]. In 3D, we estimate numerically partial volumes through a combined strategy of recursive subdivision and sampling. We also investigate the use of smooth influence functions with a finite support to improve the accuracy and convergence of numerical solutions in peridynamics.

The organization of this paper is as follows. In Section 2, we briefly review the state-based peridynamic theory, a corresponding meshfree discretization, algorithms from the literature for approximations of partial volumes, and influence functions. We study the convergence of numerical solutions for a one-dimensional static bond-based peridynamic problem in Section 3 and for a two-dimensional static bond-based peridynamic problem in Section 4. In Section 5, we present a combined recursive subdivision and sampling algorithm to estimate partial volumes in 3D, and we study the convergence of numerical solutions for a three-dimensional static state-based peridynamic problem. Concluding remarks are given in Section 6.

2. The peridynamic theory

The peridynamic (PD) theory [1,2,35,36] is a nonlocal reformulation of the classical theory of continuum solid mechanics. We refer to it as a *nonlocal* theory, because interactions between material points in the PD theory occur across finite distances; in contrast, the classical theory is *local*. Other nonlocal models in solid mechanics can be found in, e.g., [37–41].

Given a bounded body $\overline{\overline{\Omega}} \subset \mathbb{R}^d$, $d = 1, 2$, or 3, the PD equation of motion for a material point $\mathbf{x} \in \overline{\overline{\Omega}}$ at time $t \geqslant 0$ is

$$\rho(\mathbf{x})\frac{\partial^2\mathbf{u}}{\partial t^2}(\mathbf{x},t) = \int_{\overline{\overline{\Omega}}} \left\{\underline{\mathbf{T}}[\mathbf{x},t]\langle\mathbf{x}'-\mathbf{x}\rangle - \underline{\mathbf{T}}[\mathbf{x}',t]\langle\mathbf{x}-\mathbf{x}'\rangle\right\} dV_{\mathbf{x}'} + \mathbf{b}(\mathbf{x},t), \tag{1}$$