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Lattice Boltzmann simulations of a time-dependent natural convection problem

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ABSTRACT

A two-dimensional double Multiple-Relaxation-Time thermal lattice Boltzmann method is used to simulate natural convection flows in differentially heated cavities. The buoyancy effects are considered under the Boussinesq assumption. Flow and temperature fields are respectively solved with nine and five discrete velocities' models. Boundary conditions are implemented with the classical bounce-back or an "on-node" approach. The latter uses the popular Zou and He and Counter-Slip formulations. This paper evaluates the differences between the two implementations for steady and time-dependent flows as well as the space and time convergence orders.

1. Introduction

The Lattice Boltzmann Method (LBM), derived from the lattice gas automata [1], has been developed as an alternative numerical scheme for solving the incompressible Navier–Stokes equations. It has demonstrated its ability to simulate several physical systems [2]. Its straightforward implementation, natural parallelism and easy boundary condition treatment [3] make it very efficient and accurate for hydrodynamical flows. However, for thermal flows, its efficiency is a pending issue.

Indeed, thermal lattice Boltzmann methods can be split into three main classes. The first one relies on increasing the number of discrete velocities in order to make the original athermal LBM able to solve correctly the temperature field [4]. The main disadvantages of this approach are the loss of the cellular automata transport scheme and a fixed Prandtl number. The second one is the hybrid approach. Mass and momentum are solved with a lattice Boltzmann model while the convection–diffusion equation for temperature is solved with a classic macroscopic solver like Finite-Volume or Finite-Element methods [5–7]. Counterparts are a harder parallelization and the resolution of a linear system. Finally, the third one, the double distribution function formulation, uses two evolution equations: one for the mass and momentum conservation and one for the temperature [8–10]. This approach preserves the natural advantages of lattice methods: the local formulation and the explicit time evolution scheme. Besides, supplementary distribution functions can be added to take into account additional effects like magnetic field [11]. Mass and momentum equations can be solved with the generalized, or Multiple-Relaxation-Time (MRT), formulation [12,13]. Two approaches stand out for the thermal part: the internal energy formulation [14,15] and lattice Boltzmann schemes designed for convection–diffusion equations [16]. The former might be suitable for non-straight boundary condition [17] and fluid–solid conjugate heat transfer [18,19]. The latter offers more flexibility in the discrete velocities' set [20] and recent developments show that the evolution equation can be solved with an MRT collision operator [10,21–23].

In many cases, the lattice Boltzmann method is used for stationary flows and does not enjoy the benefits of the inherent unsteady formulation. Indeed, the physical transient flow has to be simulated and calculations of accurate solutions would be very expensive compared to stationary solvers.

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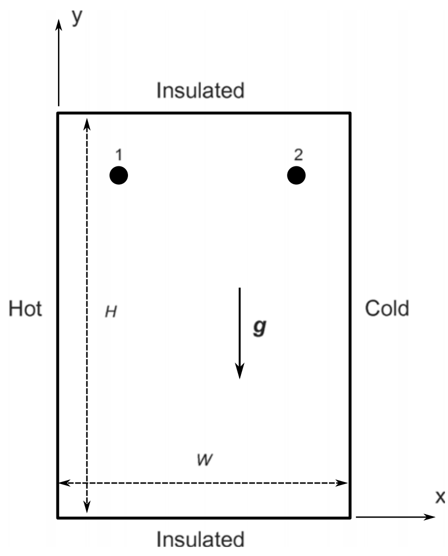

Fig. 1. Differentially heated cavity with insulated horizontal walls and constant temperature vertical walls. The aspect ratio is $A = H/W$. Points 1 and 2, respectively in (x_1, y_1) and (x_2, y_2), are time history points.

Considering this background, this work aims to analyze results and convergence orders of the double MRT method for a time-dependent natural convection flow configuration. Reference values are extracted from benchmark solutions on differentially heated cavities [24,25]. Proper comparisons with theoretical second-order accuracy both in space and time [2] are also made thanks to Richardson Extrapolation. Because of numerical instability, this analysis can hardly be handled using the classical LBM method with the Bhatnagar–Gross–Krook (BGK) collision term.

This paper is organized as follows. Section 2 presents the general configuration of the differentially heated cavity under the Boussinesq approximation. In Section 3, the numerical method is described: a double MRT lattice Boltzmann method using respectively nine and five velocities' models for the fluid flow and the energy equation is detailed. The obtained results for two configurations are presented in Section 4. Comparisons with reference and convergence orders are discussed. Finally, the last section is dedicated to the concluding remarks.

2. Problem description

The configuration studied is the natural convection in a two-dimensional cavity heated differentially on vertical side walls. The configuration is illustrated in Fig. 1 where W is the width and H the height of the cavity. The cavity aspect ratio is $A = H/W$. The gravity vector is directed in the negative y-coordinate direction. The Boussinesq hypothesis is used and only small temperature variations from the mean temperature are admitted.

The dimensionless formulation of the incompressible Navier–Stokes and energy equation coupled with the Boussinesq hypothesis for a time-dependent convection problem is written as

$$\begin{cases} \nabla \cdot \boldsymbol{u} = 0 \\ \dfrac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\nabla p + \sqrt{\dfrac{Pr}{Ra}} \nabla^2 \boldsymbol{u} + \theta \boldsymbol{e}_y \\ \dfrac{\partial \theta}{\partial t} + \boldsymbol{u} \cdot \nabla \theta = \sqrt{\dfrac{1}{RaPr}} \nabla^2 \theta \end{cases} \tag{1}$$

where $\boldsymbol{u} = (u, v)$, p and θ are respectively the velocity, pressure and temperature fields and $\boldsymbol{e}_y$ is the unit vector in the y-direction. This dimensionless system was obtained using the characteristic length W, buoyancy velocity scale $U = \sqrt{g\beta W \Delta T}$, time scale W/U and pressure scale ρU^2. Here, ρ is the mass density, g the gravitational acceleration and β the coefficient of thermal expansion. The dimensionless temperature is defined as follows:

$$\theta = \frac{T - T_{ref}}{T_h - T_c} \quad \text{with } T_{ref} = \frac{T_h + T_c}{2} \tag{2}$$

and T_h and T_c are respectively the prescribed temperatures of hot and cold walls. The Rayleigh and Prandtl numbers are control parameters of the problem and are written as

$$Ra = \frac{g\beta \Delta T W^3}{\nu\alpha} \quad \text{and} \quad Pr = \frac{\nu}{\alpha} \tag{3}$$