Computers and Mathematics with Applications 66 (2013) 1372–1388

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

# A generalized product-type BiCOR method and its application in signal deconvolution ☆

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ARTICLE INFO

*Article history:*
Received 12 June 2012
Received in revised form 4 March 2013
Accepted 3 August 2013

*Keywords:*
Krylov subspace method
Product-type method
BiCOR
Signal deconvolution

ABSTRACT

For solving nonsymmetric linear systems, we attempt to establish symmetric structures in nonsymmetric systems and handle them through the methods devised for symmetric cases. A Biconjugate $A$-Orthogonal Residual method based on Biconjugate $A$-Orthonormalization Procedure has been proposed and nominated as BiCOR in [Y.-F. Jing, T.-Z. Huang, Y. Zhang, L. Li, G.-H. Cheng, Z.-G. Ren, Y. Duan, T. Sogabe, B. Carpentieri, Lanczos-type variants of the COCR method for complex nonsymmetric linear systems, J. Comput. Phys. 228 (2009) 6376–6394.]. As many similar characteristics exist between BiCOR and BiCG, the strategies of improved variants of BiCG, such as CGS and BiCGSTAB, can be utilized to enhance the algorithm for BiCOR. Making use of the product of residual polynomials of BiCOR and other polynomials, CORS and BiCORSTAB have been proposed along the same ideas of CGS and BiCGSTAB, respectively in the above-mentioned paper. In this paper, a unified generalized framework of product-type BiCOR, which is epitomized by the product of residual polynomials and other polynomials, is proposed. Numerical examples are selected from the blurring signal cases and the effect of the generalized product-type BiCOR method is prominent in signal deconvolution.



## 1. Introduction

Algorithms for solving symmetric linear systems, such as the Conjugate Gradient (CG) method [1] and the Conjugate Residual (CR) method [2], are relatively efficient due to cheaper storage. That is, we only need to keep recent approximate solutions and relevant parameters for next iterations. Hence, we expect to establish symmetric structures in nonsymmetric linear systems so that we can utilize the advantages of algorithms for symmetric cases. The Biconjugate Gradient (BiCG) method [3,4] is constructed based on the biorthogonalization process and the *Petrov–Galerkin* approach (the definition of such approach locates in [5]), and we can derive the algorithm of BiCG depending on the symmetric structure and a dual linear system with details in [3,4]. The method makes it possible that we need $O(n)$ extra storage, instead of $O(nk)$ in the Generalized Minimal Residual (GMRES) method [6] beside that for the coefficient matrix, and performs $O(n)$ operations during algorithm implementation. A number of numerical experiments have demonstrated that BiCG is a competitive method for nonsymmetric linear systems.

Considering the differences and similarities between CG and CR, a kind of CR derived methods based on the biorthogonalization process and the *Petrov–Galerkin* approach has been proposed by Sogabe, Sugihara and Zhang [7], Jing,

☆ This research is supported by NSFC (61170311,11201055), 973 Program (2013CB329404), Chinese Universities Specialized Research Fund for the Doctoral Program (20110185110020), and Sichuan Province Sci. & Tech. Research Project (2012GZX0080).

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http://dx.doi.org/10.1016/j.camwa.2013.08.007

Huang, et al. [8], and Carpentieri, Jing and Huang [9]. It is nominated as the Biconjugate $A$-Orthogonal Residual (BiCOR) method. Similar to BiCG, BiCOR suffers from a matrix–vector multiplication with both $A$ and $A^T$ at each iteration, which exacerbates the amount of computation and storage. Taking into account recursive form in BiCOR, a product of two polynomials to replace the residual polynomial can be used so that the computation of matrix–vector multiplication by $A^T$ can be avoided. Like the Conjugate Gradient squared (CGS) method [10], constructing the residual polynomial as the square of residual polynomial in BiCG, the Conjugate $A$-Orthogonal Residual squared (CORS) has been established along a similar idea to CGS [8,9]. Another well-known variant of BiCG, named the Biconjugate Gradient stabilized (BiCGSTAB) method [5,11], utilizes a product of an auxiliary polynomial and the residual polynomial of BiCG to displace residual polynomial, where the auxiliary polynomial is constructed in order to reach the minimum 2-norm of residual at each iteration. According to the same principle in choosing the auxiliary polynomial, the Biconjugate $A$-Orthogonal Residual stabilized (BiCORSTAB) method [8], a variant of BiCOR, has been established. The variants of BiCOR can be combined with preconditioners, when suffering from ill-conditioned problems, to accelerate convergence and relevant algorithms have been given in [8]. But this case is outside of the scope of this paper and we do not focus on it here.

Taking the structure of the residual polynomials in CGS and BiCGSTAB into consideration, we can summarize them as product-type method that is replacing the residual polynomials by a product of residual polynomial and a certain other polynomial (or we denote it as an auxiliary polynomial in this paper). Zhang has proposed a method named Generalized product-type methods based Bi-CG (GPBiCG) [12] as a generalized framework to illustrate the product-type based variants of BiCG without enlargement both on computation and storage.

In this paper, we propose an iterative Krylov method based on BiCOR, denoted as generalized product-type BiCOR, to illustrate the variants, such as CORS and BiCORSTAB, in an unified generalized framework. CORS and BiCORSTAB can both be regarded as a special manifestation of the generalized framework. Hence, we can search for more efficient auxiliary polynomials to improve BiCOR under this framework, not limiting within CORS and BiCORSTAB. In addition, generalized product-type BiCOR performs well in signal deconvolution and numerical experiments will be presented to verify the statement.

The remainder of this paper is organized as follows. In Section 2, BiCOR and relevant information will be reviewed first. CORS and BiCORSTAB will be re-stated in Section 3. In Section 4, we deduce the generalized product-type BiCOR and give the relevant algorithm. The numerical experiment results will be stated in Section 5 and we summarize this paper in the last section.

## 2. Biconjugate *A*-Orthonormalization and BiCOR

The BiCOR method is an iterative algorithm for solving real nonsymmetric or complex non-Hermitian linear systems [8,9]. Given an initial guess $x_0$, for solving the following linear system

$$Ax = b, \quad A \in R^{n\times n},\ b \in R^n, \tag{1}$$

popular iterative Krylov methods, such as GMRES [6] and CG [1], all aim to extracting an approximate solution in the affine subspace, i.e., $x_0 + K_m$, where $K_m$ is formed as

$$K_m(A, r_0) = \operatorname{span}\{r_0, Ar_0, A^2r_0, \dots, A^{m-1}r_0\}, \tag{2}$$

and $r_0$ is the initial residual which is obtained as $r_0 = b - Ax_0$. To define the ultimate approximate solution, we need to impose certain Krylov subspace approach [5]. GMRES is based on the minimum residual norms, that is the $m$th approximation $x_m$ satisfies to minimize $\|b - Ax_m\|_2$ over $K_m$, and CR is an algorithm derived for Hermitian cases. There is a class of methods that are dependent on the *Petrov–Galerkin* approach, under which we construct the approximate solution $x_m$ so that the residual $r_m$ is orthogonal to another subspace, i.e., $b - Ax_m \perp L_m$. For illustrating BiCOR, we review the biconjugate $A$-Orthonormalization process [8] firstly.

### 2.1. The biconjugate A-Orthonormalization process

Recalling the Gram–Schmidt orthonormalization process, we select an arbitrary vector as the initial basis, and construct next basis vector orthogonal to previous basis vectors. With $k$ steps of Gram–Schmidt process, a set of basis vectors of a $k$-degree subspace is obtained.

Summarizing briefly, the biconjugate $A$-Orthonormalization process [8] performs as follows: 1. Choose two arbitrary vectors such that $(w_1, Av_1) = 1$ where $(\cdot,\cdot)$ stands for a standard dot product and similarly hereinafter; 2. Multiplied by $A$, the recent $v_k$ is orthogonal to $w_i$'s where $i < k$, and the recent $w_k$, multiplied by $A^T$, is orthogonal against $v_i$'s where $i < k$. To reach a symmetric structure similar to the one in CR, the following three-term recurrence relations have been presented as in Algorithm 1 of [8]

$$\delta_i v_i = Av_{i-1} - \beta_{i-1}v_{i-2} - \alpha_{i-1}v_{i-1},$$
$$\beta_i w_i = A^T w_{i-1} - \delta_{i-1}w_{i-2} - \alpha_{i-1}w_{i-1}.$$