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RESEARCH PAPER

Stress interference calculation model and its application in volume stimulation of horizontal wells

XU Yun[1, 2], CHEN Ming[1, 3,*], WU Qi[2, 4], LI Deqi[5], YANG Nengyu[4], WENG Dingwei[1, 2], GUAN Baoshan[1, 2]

1. Langfang Branch of PetroChina Research Institute of Petroleum Exploration & Development, Langfang 065007, China;
2. The Key Laboratory of Research Stimulation, PetroChina, Langfang 065007, China;
3. PetroChina Research Institute of Petroleum Exploration & Development, Beijing 100083, China;
4. PetroChina Exploration and Production Company, Beijing 100007, China;
5. PetroChina Zhejiang Oilfield Company, Hangzhou 310023, China

Abstract: A new model for calculating stress fields of fractured media was established by incorporating stress correction factor based on displacement discontinuity boundary element method. The accuracy of the new model is close to 3D displacement discontinuity model, and its calculation is significantly simplified. An algorithm for multi-fracture propagation geometry was proposed based on fracture criterion and fracture growth rate law, which was used to investigate multi-fracture stress interference and propagation geometry. The results show that the size of stress interference is determined by the shortest dimension of fracture face, which is 1.2–1.5 times fracture height when fracture length is longer than fracture height, and 1.2–1.5 times fracture length when fracture length is shorter than fracture height. The larger the ratio of fracture spacing to fracture height, or the smaller the ratio of net pressure to the differential principle stress, the more close to well-bores the deviation position is, and the larger the deviation angle is. The middle fracture propagates to the fracture at a further distance and one dominating fracture propagates longest when three-cluster fractures are not equally spaced, while the middle fracture propagates straightly when three-cluster fractures are equally spaced.

Key words: horizontal well; volume fracturing; stress interference; displacement discontinuity method; fracture deviation; multi-fracture propagation; unequally distributed fracture

Introduction

Stress interference, also termed as induced stress or stress shadowing effect in hydraulic fracturing, is the perturbation of in situ stress induced by fracture opening or sliding[1, 2]. Getting a clear understanding on the action rules of stress interference and multi-fracture propagation paths under the influence of stress interference is of great significance for optimizing design and production forecast of volume fracturing[3, 4].

Sneddon[5], Green[6] and Pollard[7] deduced the analytical solution of plane strain fracture stress fields, which only applies to 2D straight fractures rather than complex fractures. The calculation of stress fields induced by complex fractures required numerical techniques, such as FEM (finite element method), FDM (finite difference method), FVM (finite volume method), DEM (discrete element method) and BEM (boundary element method), among them, the first four need to discrete the whole region to get the numerical solution, while the boundary element method only needs to discrete the boundary by solving the boundary integral equation, which significantly reduces the calculation amount. Therefore, BEM is more suitable for problems about fractures or fracture mechanics in infinite medium[8, 9]. Crouch[10] proposed 2D DDM boundary element method in 1976, which was extended to 3D DDM boundary element method by introducing asymptotic solution of fracture tip by Shou[11] in 1993, but the calculation amount increased significantly. Olson[12] deduced a pseudo 3D DDM boundary element method, but neglecting the handling of the calculated fracture width. The unconventional fracturing model (UFM) proposed by Weng Xiaowei et al[13]. and the fracture propagation model established by Wu Kan et al[14]. adopted Olson's model to describe rock deformation, which may influence treatment decisions because of the model's calculation errors.

In view of the deficiency of stress calculation model, a new

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* **Corresponding author.** E-mail: xmcm0122@126.com
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calculation model for stress field of fractured medium has been proposed based on 2D DDM boundary element method by incorporating stress correction factor, and compared with 3D DDM model to verify its reliability. By solving multi-fracture growth rate explicitly based on fracture mechanics, an algorithm for calculating multiple fracture propagation has been advanced, and the factors affecting stress interference and multi-fracture propagation under stress interference have been investigated, and finally a case has been examined by the model.

1. Model

1.1. Model building

DDM is a numerical method based on the analytical solution of a single elemental displacement discontinuity, which is similar in principle to Green Function method used in fluid mechanics. For a 3D nonplanar hydraulic fracture with fracture length L and fracture height H (Fig. 1), the fracture is divided into N elements of equal size along fracture length. The half length of each element is a, the central point co-ordinates of an element are(x_i , y_i), i=1, 2,..., N. The displacement discontinuity between the two sides of the element is defined as follows[10]:

$$\begin{cases} D_{\mathrm{s}} = u_{\mathrm{s}}\left(s, 0^-\right) - u_{\mathrm{s}}\left(s, 0^+\right) \\ D_{\mathrm{n}} = u_{\mathrm{n}}\left(s, 0^-\right) - u_{\mathrm{n}}\left(s, 0^+\right) \end{cases} \quad \left(|s| \le a\right) \tag{1}$$

Based on the analytical solution of a constant displacement discontinuity[10], the stresses in the s-n co-ordinate system induced by displacement discontinuity of j element are

$$\begin{cases} \sigma_{ss} = 2G\left(2f_{sn} + nf_{snn}\right)D_{sj} + 2G\left(f_{sn} + nf_{nnn}\right)D_{\mathrm{n}j} \\ \sigma_{\mathrm{n}s} = 2G\left(f_{nn} + nf_{snn}\right)D_{sj} - 2Gnf_{snn}D_{\mathrm{n}j} \\ \sigma_{\mathrm{nn}} = -2Gnf_{snn}D_{sj} + 2G\left(f_{nn} - nf_{nnn}\right)D_{\mathrm{n}j} \end{cases} \tag{2}$$

$$f\left(s,n\right) = -\frac{1}{4\pi\left(1-\nu\right)}\left[n\left(\arctan\frac{n}{s-a} - \arctan\frac{n}{s+a}\right) - \left(s-a\right)\ln\sqrt{\left(s-a\right)^2 + n^2} + \left(s+a\right)\ln\sqrt{\left(s+a\right)^2 + n^2}\right] \tag{3}$$

The coordinates of the central point of i element in the co-ordinate system of j element are

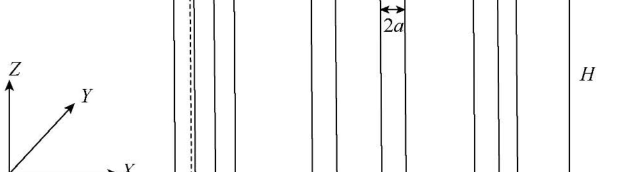

Fig. 1. Co-ordinates and sketch of a fracture.

$$\begin{cases} s_{ij} = \left(x_i - x_j\right)\cos\theta_j + \left(y_i - y_j\right)\sin\theta_j \\ n_{ij} = -\left(x_i - x_j\right)\sin\theta_j + \left(y_i - y_j\right)\cos\theta_j \end{cases} \tag{4}$$

On the basis of stress coordinate transformation[15], the stresses in i element caused by the displacement discontinuity $D_{\mathrm{s}j}$, $D_{\mathrm{n}j}$ are

$$\begin{cases} \sigma_{\mathrm{n}ij} = 2G\left[-f_{ss} + n_{ij}\sin\left(2\gamma_{ij}\right)f_{snn} - n_{ij}\cos\left(2\gamma_{ij}\right)f_{nnn}\right]D_{\mathrm{n}j} + \\ \quad 2G\left[2\sin^2\left(\gamma_{ij}\right)f_{sn} + \sin\left(2\gamma_{ij}\right)f_{sn} - n_{ij}\cos\left(2\gamma_{ij}\right)f_{snn} - \right. \\ \quad \left. n_{ij}\cos\left(2\gamma_{ij}\right)f_{nnn}\right]D_{\mathrm{s}j} \\ \sigma_{\mathrm{s}ij} = 2G\left[-n_{ij}\cos\left(2\gamma_{ij}\right)f_{snn} - n_{ij}\sin\left(2\gamma_{ij}\right)f_{nnn}\right]D_{\mathrm{n}j} + \\ \quad 2G\left[-\sin\left(2\gamma_{ij}\right)f_{sn} - \cos\left(2\gamma_{ij}\right)f_{sn} - n_{ij}\sin\left(2\gamma_{ij}\right)f_{snn} + \right. \\ \quad \left. n_{ij}\cos\left(2\gamma_{ij}\right)f_{nnn}\right]D_{\mathrm{s}j} \end{cases} \tag{5}$$

where $\gamma_{ij}=\theta_i-\theta_j$

The results in equation (5) can be simply rewritten in the form

$$\begin{cases} \sigma_{\mathrm{n}ij} = C_{\mathrm{nn}ij}D_{\mathrm{n}j} + C_{\mathrm{ns}ij}D_{\mathrm{s}j} \\ \sigma_{\mathrm{s}ij} = C_{\mathrm{sn}ij}D_{\mathrm{n}j} + C_{\mathrm{ss}ij}D_{\mathrm{s}j} \end{cases} \tag{6}$$

where

$$C_{\mathrm{nn}ij} = 2G\left[-f_{ss} + n_{ij}\sin\left(2\gamma_{ij}\right)f_{snn} - n_{ij}\cos\left(2\gamma_{ij}\right)f_{nnn}\right]$$

$$C_{\mathrm{ns}ij} = 2G\left[2\sin^2\left(\gamma_{ij}\right)f_{sn} + \sin\left(2\gamma_{ij}\right)f_{sn} - n_{ij}\cos\left(2\gamma_{ij}\right)f_{snn} - n_{ij}\cos\left(2\gamma_{ij}\right)f_{nnn}\right]$$

$$C_{\mathrm{sn}ij} = 2G\left[-n_{ij}\cos\left(2\gamma_{ij}\right)f_{snn} - n_{ij}\sin\left(2\gamma_{ij}\right)f_{nnn}\right]$$

$$C_{\mathrm{ss}ij} = 2G\left[-\sin\left(2\gamma_{ij}\right)f_{sn} - \cos\left(2\gamma_{ij}\right)f_{sn} - n_{ij}\sin\left(2\gamma_{ij}\right)f_{snn} + n_{ij}\cos\left(2\gamma_{ij}\right)f_{nnn}\right]$$

The stresses in element i induced by all N displacement discontinuities based on superposition principle are

$$\begin{cases} \sigma_{\mathrm{n}i} = \sum_{j=1}^{N} C_{\mathrm{nn}ij}D_{\mathrm{n}j} + \sum_{j=1}^{N} C_{\mathrm{ns}ij}D_{\mathrm{s}j} \\ \sigma_{\mathrm{s}i} = \sum_{j=1}^{N} C_{\mathrm{sn}ij}D_{\mathrm{n}j} + \sum_{j=1}^{N} C_{\mathrm{ss}ij}D_{\mathrm{s}j} \end{cases} \quad \left(i = 1, 2, \cdots, N\right) \tag{7}$$

The above displacement discontinuity calculation model is based on analytical solution of equation (2), in which the height effect on stress influence coefficients doesn't take into consideration. In the paper, to avoid increasing computation complexity, the influence of element or fracture height was considered by modifying the stress influence coefficients in equation (2). For a plane fracture with height H, the stress perpendicular to fracture face or along y axis is[16]

$$\sigma_{yy} = p_{\mathrm{n}}\left[1 - \frac{|y|^3}{\left(y^2 + H^2\right)^{3/2}}\right] \tag{8}$$

Following the form of equation (8), the stress correction factor can be expressed as