PETROLEUM EXPLORATION AND DEVELOPMENT
Volume 43, Issue 3, June 2016
Online English edition of the Chinese language journal

Available online at www.sciencedirect.com
ScienceDirect

Cite this article as: PETROL. EXPLOR. DEVELOP., 2016, 43(3): 506–513.

RESEARCH PAPER

Through-wall yield collapse pressure of casing based on unified strength theory

LIN Yuanhua[1,*], DENG Kuanhai[1], SUN Yongxing[2], ZENG Dezhi[3], XIA Tianguo[4]

1. State Key Laboratory of Oil and Gas Reservoir Geology and Exploitation (Southwest Petroleum University), Chengdu 610500, China;
2. CNPC Key Lab for Tubular Goods Engineering (Southwest Petroleum University), Chengdu 610500, China,
3. CCDC Drilling&Production Technology Research Institute 618300, Zhongshan Avenue South 2nd Section,Guanhan city,Sichuan Province.
4. Tarim Oil Field Company, CNPC, Korla, Xinjiang 841000, China.

Abstract: The unified algorithm of through-wall yield collapse pressure for casing with due consideration of strength differential (SD), yield-to-tensile strength ratio, material hardening and intermediate principal stress, which is suitable to calculate collapse strength of all casing has been obtained based on unified strength theory, and four classical through-wall yield collapse formulas of casing have been presented based on the L.Von Mises, TRESCA, GM and twin yield strength criterion. The calculated value is maximum based on the twin yield strength criterion, which can be used as upper limit of through-wall yield collapse pressure, and the calculated value is minimum based on the TRESCA strength criterion, which can be used as lower limit of through-wall yield collapse pressure in the design process. Numerical and experimental comparisons show that the equation proposed by this paper is much closer to the collapse testing values than that of other equations.

Key words: unified strength theory; casing collapse; through-wall yield; collapse pressure; strength differential; intermediate principal stress

Introduction

In recent years, with the deepening of oil and gas exploration and development, deep wells and ultra-deep wells[1–2] and wells encountering complex formations such as mudstone, shale and plastic rock salt stratum are increasing too, in these wells, casing is exposed in more complicated and harsh conditions, and subjected to higher external pressure in deep and super-deep wells in particular, so conventional API casing cannot meet the strength requirement of deep wells and ultra-deep wells, restricting the development of drilling technology for and drilling depth extension of deep wells[3].

In order to meet the strength requirement on casing in deep and ultra-deep wells, many kinds of heavy wall casings, non-API casings and high collapse casings have been developed successively[4–6], which have higher anti-collapsing capacity than conventional API casings[7]. Even so, there are still cases of failure of high strength casings in deep and ultra-deep wells reported in recent years[8]. Hence, the collapse strength prediction and design of casing are very important for ensuring the safety of oil and gas production.

To predict and design the collapse strength of casing accurately, many researches on the casing collapse strength under uniform load[9–14] have been conducted in China and abroad, and some important results (such as building some classical models) have been achieved[12, 14]. But the design principles of most of those mechanical models are the initial yield of inner wall of casing under external pressure. In fact, the casing is not damaged completely and still has a significant amount of remaining anti-collapse capacity when the inner wall yields[15–16]. Many tubular products would be wasted in conventional wells and the selection of casing would be difficult if following this principle. In addition, most of the mechanical models[10–12] were built based on the classical Tresca yield criterion and von Mises yield criterion without due consideration of the effect of SD effect and intermediate principal stress on the yield collapse pressure of casing, which makes those mechanical models only applicable to materials with equal tensile and collapse strength[17]. Moreover, further study shows that yield collapse formula in ISO 10400 collapse model[14] is not a complete through-wall yield collapse formula, which underestimates the through-wall yield collapse pressure of casing, and makes the ISO 10400 model not suitable for predicting the collapse strength of casing of all sizes[8, 16].

Hence, a collapse formula considering SD effect, intermediate principal stress, material hardening and yield-to-tensile strength ratio, based on unified strength theory used widely in engineering field, which can predict through-wall yield collapse pressure of casing, has been presented in this paper[18–20].

Received date: 08 Apr. 2015; **Revised date:** 09 Mar. 2016.

*** Corresponding author.** E-mail: dengkuanhai@163.com

Foundation item: Supported by the National Natural Science Foundation of China (51274170).

1. Through-wall yield collapse formula for casing

1.1. Unified strength theory

Many existing strength criteria and some new strength criteria can be derived from the unified strength theory[18–19] which takes the effects of intermediate principal stress and SD effect of material on the material strength into account, and can be used to analyze plastic limit of various materials. The unified strength theory can be given as follows:

$$\begin{cases} f=\sigma_1-\dfrac{\alpha}{1+b}(b\sigma_2+\sigma_3)=\sigma_\mathrm{s} & \sigma_2\le\dfrac{\sigma_1+\alpha\sigma_3}{1+\alpha} \\ f'=\dfrac{1}{1+b}(\sigma_1+b\sigma_2)-\alpha\sigma_3=\sigma_\mathrm{s} & \sigma_2\ge\dfrac{\sigma_1+\alpha\sigma_3}{1+\alpha} \\ \alpha=\dfrac{\sigma_\mathrm{t}}{\sigma_\mathrm{c}} & 0\le\alpha\le1 \\ b=\dfrac{(1+\alpha)\tau_\mathrm{s}-\sigma_\mathrm{t}}{\sigma_\mathrm{t}-\tau_\mathrm{s}} & 0\le b\le1 \end{cases} \tag{1}$$

1.2. Elastic limit analysis of casing under uniform external pressure

The stress components (σ_θ, σ_r, σ_z) of the casing increase with the increase of external pressure (p_o). The casing will be in the elastic limit state, when the inner wall yields initially. Assume that the casing is a long thick-walled cylinder. The long thick-walled cylinder is in the elastic state, when the external pressure (p_o) is lower. According to the Lame formula[17], the following stress components can be obtained:

$$\begin{cases} \sigma_r=-\dfrac{R_\mathrm{o}{}^2p_\mathrm{o}}{R_\mathrm{o}{}^2-R_\mathrm{i}^2}\left(1-\dfrac{R_\mathrm{i}^2}{R^2}\right) \\ \sigma_\theta=-\dfrac{R_\mathrm{o}{}^2p_\mathrm{o}}{R_\mathrm{o}{}^2-R_\mathrm{i}^2}\left(1+\dfrac{R_\mathrm{i}^2}{R^2}\right) \\ \sigma_z=\dfrac{m}{2}(\sigma_r+\sigma_\theta) \end{cases} \tag{2}$$

The mechanical analysis of long thick-walled cylinder under external pressure belongs to the axisymmetric plane strain problem. Hence, according to the study on the plane strain problem[21–22], the plasticity coefficient m (m=2v in the elastic region, m=1 in the plastic region) can be obtained. When the inner wall is in the elastic limit state (in the plastic region), by the Eq. (2), Eq. (3) can be obtained:

$$\sigma_z=\frac{m}{2}(\sigma_r+\sigma_\theta)=-\frac{R_\mathrm{o}{}^2p_\mathrm{o}}{R_\mathrm{o}{}^2-R_\mathrm{i}{}^2} \tag{3}$$

According to the stress state ($\sigma_r\ge\sigma_z\ge\sigma_\theta$) of thick-walled cylinder, it can be known that the first, second and third principal stress (σ_1, σ_2 and σ_3) are equal to σ_r, σ_z and σ_θ respectively. The second principal stress (σ_2) is not more than $(\sigma_1+\alpha\sigma_3)/(1+\alpha)$ because the tensile strength-compressive strength ratio (α) is less than 1.0. By substituting the Eq. (2), Eq. (3) into Eq. (1), the Eq. (4) can be obtained:

$$-\frac{R_\mathrm{o}{}^2p_\mathrm{o}}{R_\mathrm{o}{}^2-R_\mathrm{i}{}^2}\left[\left(1-\frac{R_\mathrm{i}{}^2}{R^2}\right)-\frac{b\alpha}{2(1+b)}\left(1-\frac{R_\mathrm{i}{}^2}{R^2}\right)-\frac{\alpha}{2}\left(1+\frac{R_\mathrm{i}{}^2}{R^2}\right)-\frac{\alpha}{2(1+b)}\left(1+\frac{R_\mathrm{i}{}^2}{R^2}\right)\right]=\sigma_\mathrm{s} \tag{4}$$

The inner wall (R=R_i) of thick-walled cylinder yields firstly, when the external pressure p_o is equal to the elastic limit pressure (p_y). By the Eq. (4), the elastic limit pressure (p_y) can be obtained:

$$p_\mathrm{y}=\frac{(1+b)(R_\mathrm{o}{}^2-R_\mathrm{i}^2)\sigma_\mathrm{s}}{\alpha(2+b)R_\mathrm{o}{}^2} \tag{5}$$

1.3. Plastic limit analysis of casing under external pressure

When p_o>p_y, the plastic area will be formed near the inner wall of the thick-walled cylinder. Assume that R_c is the radius of interface between elastic region and plastic region. The plastic area will extend from inner surface to outer surface with the increase of external pressure, so the range of $R_\mathrm{i}\le R\le R_\mathrm{c}$ is plastic area and the range of $R_\mathrm{c}\le R\le R_\mathrm{o}$ is elastic area, as shown in Fig. 1. The interface between the elastic area and plastic area is a cylinder surface due to the axial symmetry of stress components (σ_θ and σ_r), and the interfacial pressure p_i is applied to the interface. Therefore, the outer cylinder (elastic area) and inner cylinder (plastic area) can be also analyzed based on the principle of long thick-walled cylinder. Based on axial symmetry, the interfacial pressure (p_i) and external pressure (p_o) act on inner wall and outer wall of outer cylinder respectively, and only interfacial pressure (p_i) acts on outer wall of the inner cylinder.

For the inner cylinder (plastic region $R_\mathrm{i}\le R\le R_\mathrm{c}$), according to the equilibrium equation of plastic mechanics, the Eq. (6) can be obtained:

$$\frac{\mathrm{d}\sigma_r}{\mathrm{d}R}+\frac{\sigma_r-\sigma_\theta}{R}=0 \tag{6}$$

By the Eq. (2) and Eq. (4), the relationship between stress component ($\sigma_r-\sigma_\theta$) and yield stress (σ_s) can be obtained, and the Eq. (7) can be given:

$$\sigma_r-\sigma_\theta=\frac{2(1+b)(\alpha-1)}{\alpha(2+b)}\sigma_r+\frac{2(1+b)}{\alpha(2+b)}\sigma_\mathrm{s} \tag{7}$$

Substituting Eq. (7) into Eq. (6) gives:

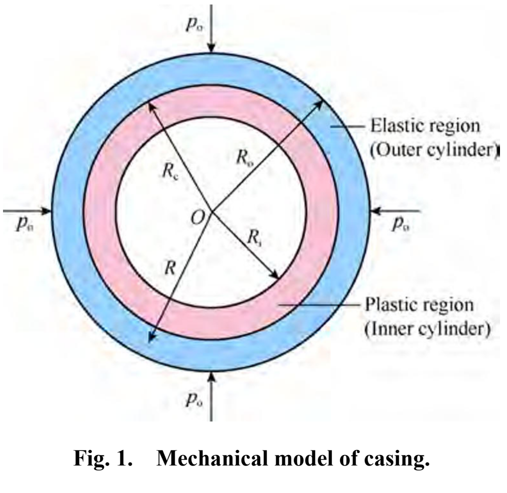

Fig. 1. Mechanical model of casing.