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Analytic study on a (2 + 1)-dimensional nonlinear Schrödinger equation in the Heisenberg ferromagnetism

De-Yin Liu, Bo Tian *, Yan Jiang, Xi-Yang Xie, Xiao-Yu Wu

State Key Laboratory of Information Photonics and Optical Communications, and School of Science, Beijing University of Posts and Telecommunications, Beijing 100876, China

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ABSTRACT

In this paper, a (2 + 1)-dimensional nonlinear Schrödinger equation for a (2 + 1)-dimensional Heisenberg ferromagnetic spin chain with the bilinear and anisotropic interactions is investigated. Via the Hirota method and symbolic computation, bilinear forms and multi-soliton solutions are derived. The one, two and three solitons are analyzed graphically and we find the amplitudes and widths of the two and three solitons keep invariant after each interaction. The bell-shape one soliton as well as parallel, crossed two and three solitons are respectively observed. Through the asymptotic analysis, expressions which denote the two solitons before and after the interactions are obtained and interactions between the two solitons are proved to be elastic.

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1. Introduction

Nonlinear phenomena in optics, plasma physics, condensed matter physics and other fields can be described by the corresponding nonlinear evolution equations (NLEEs) [1–4], through which we can analyze the underlying dynamics. As the solutions of the NLEEs, solitons have been studied in those fields [5–8]. Solitons are the stable localized waves that propagate in a nonlinear medium without amplitude attenuation and shape change due to the balance between the dispersion and nonlinearity [9–11]. Solitons may be either bright or dark depending on the details of the governing NLEE. A bright soliton is a peak in the amplitude; a dark soliton is a notch with a characteristic phase step across it [11].

The Heisenberg models of ferromagnetic spin chains with different magnetic interactions in the classical and semiclassical continuum limits have been connected with the NLEEs exhibiting integrability properties including the soliton spin excitations [12–15]. Nonlinear spin excitations in the magnetic materials have their applications in the microwave communication systems and nonlinear signal processing devices [16,17]. Dynamics of the nonlinear spin excitations in the Heisenberg ferromagnetism can be described by the nonlinear Schrödinger-type equations [18]. For example, a generalized perturbed fourth-order NLS equation has been proposed to describe the effect of twist inhomogeneity on the soliton spin excitations in a one-dimensional inhomogeneous helimagnet in the semiclassical limit [19]; nonlinear dynamics of the (2 + 1) dimensional ferromagnetic spin systems with bilinear and biquadratic interactions in the semiclassical limit has been investigated and the dynamics has been found to be governed by the integrable nonlinear Schrödinger (NLS) equations in (2 + 1) dimensions [20–24]. For the Heisenberg ferromagnetic spin chains, the solitary waves have been studied both theoretically and experimentally [12–15,23]. In this paper, we will consider a (2 + 1)-dimensional NLS equation [24],

$$iu_t - iu_x + u_{xx} + u_{yy} - 2u_{xy} + 2|u|^2u = 0, \qquad (1)$$

* Corresponding author.
E-mail address: tian_bupt@163.com (B. Tian).

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for the (2 + 1)-dimensional Heisenberg ferromagnetic spin chain with the bilinear and anisotropic interactions in the semiclassical limit, where $u(x, y, t)$ is a complex function representing the Heisenberg ferromagnetic spin chain, x, y and t respectively denote the scaled spatial and time coordinates, and the subscripts denote the partial derivatives. Integrability of Eq. (1) has been studied through the construction of the Lax pair and soliton solutions via the Darboux transformation [24].

However, to our knowledge, bilinear forms and soliton interactions of Eq. (1) have not been studied. In Section 2, via the Hirota method [25] and symbolic computation [26–28], we will obtain the bilinear forms and construct the one-, two- and three-soliton solutions of Eq. (1). In Section 3, we will analyze the soliton interactions via the asymptotic analysis. Figures will be plotted to illustrate the interactions of the solitons. Section 4 will be our conclusions.

2. Bilinear forms and soliton solutions

2.1. Bilinear forms

With the dependent variable transformation [25],

$$u(x, y, t) = \frac{g(x, y, t)}{f(x, y, t)}.$$

Eq. (1) can be transformed into

$$\frac{iD_t g\cdot f}{f^2} - \frac{iD_x g\cdot f}{f^2} + \frac{D_x^2 g\cdot f}{f^2} - \frac{g}{f}\frac{D_x^2 f\cdot f}{f^2} + \frac{D_y^2 g\cdot f}{f^2}$$
$$- \frac{g}{f}\frac{D_y^2 f\cdot f}{f^2} - 2\frac{D_xD_y g\cdot f}{f^2} + \frac{g}{f}\frac{D_xD_y f\cdot f}{f^2} + 2\frac{g}{f}\frac{|g|^2}{f^2} = 0.$$

Therefore, we can obtain the bilinear forms of Eq. (1) as follows:

$$(iD_t - iD_x + D_x^2 + D_y^2 - 2D_xD_y)g\cdot f = 0, \tag{2a}$$

$$(D_x^2 + D_y^2 - 2D_xD_y)f\cdot f - 2gg^* = 0, \tag{2b}$$

where $g(x, y, t)$ is the complex function of x, y and t, $f(x, y, t)$ is the real one, "$*$" denotes the complex conjugate, while D_x, D_y and D_t are all the bilinear derivative operators [25] defined by

$$D_x^m D_t^n\, a(x, t)\cdot b(x, t) = \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'}\right)^m \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)^n a(x, t)b(x', t')\bigg|_{x'=x,t'=t},$$

with $a(x, t)$, $b(x, t)$ as the differentiable functions, x', t' as the independent variables and m, n as the non-negative integers.

2.2. Soliton solutions

Based on Bilinear Forms (2), we will construct the soliton solutions of Eq. (1) by expanding $g(x, y, t)$, $f(x, y, t)$ as

$$g(x, y, t) = \epsilon g_1(x, y, t) + \epsilon^3 g_3(x, y, t) + \epsilon^5 g_5(x, y, t) + \cdots, \tag{3a}$$

$$f(x, y, t) = 1 + \epsilon^2 f_2(x, y, t) + \epsilon^4 f_4(x, y, t) + \cdots, \tag{3b}$$

where $g_j(x, y, t)$'s ($j = 1, 3, 5, \ldots$) are the complex functions with respect to x, y and t, $f_l(x, y, t)$'s ($l = 2, 4, 6, \ldots$) are the real ones and ϵ is a formal parameter.

Substituting Expressions (3), truncated as $g(x, y, t) = \epsilon g_1(x, y, t)$ and $f(x, y, t) = 1 + \epsilon^2 f_2(x, y, t)$, into Bilinear Forms (2), we obtain the one-soliton solutions as

$$u(x, y, t) = \frac{g_1(x, y, t)}{1 + f_2(x, y, t)} = \frac{e^{\theta_1}}{1 + \alpha_1 e^{\theta_1 + \theta_1^*}}, \tag{4}$$

where

$$\theta_1 = k_1 x + l_1 y + \omega_1 t, \quad \omega_1 = k_1 + i(k_1 - l_1)^2,$$
$$\alpha_1 = \frac{1}{(k_1 - l_1 + k_1^* - l_1^*)^2},$$

with k_1 and l_1 being the complex constants.

To obtain the two-soliton solutions, we truncate Expressions (3) as

$$g(x, y, t) = \epsilon g_1(x, y, t) + \epsilon^3 g_3(x, y, t), \tag{5a}$$

$$f(x, y, t) = 1 + \epsilon^2 f_2(x, y, t) + \epsilon^4 f_4(x, y, t), \tag{5b}$$

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