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A new variational approach for restoring images with multiplicative noise

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This paper proposes a novel variational model for restoration of images corrupted with multiplicative noise. It combines a fractional-order total variational filter with a high-order PDE (Laplacian) norm. The combined approach is able to preserve edges while avoiding the blocky-effect in smooth regions. This strategy minimizes a certain energy subject to a fitting term derived from a maximum a posteriori (MAP). Semi-implicit gradient descent scheme is applied to efficiently finding the minimizer of the proposed functional. To improve the numerical results, we opt for an adaptive regularization parameter selection procedure for the proposed model by using the trial-and-error method. The existence and uniqueness of a solution to the proposed variational model is established. In this study parameter dependence is also discussed. Experimental results demonstrate the effectiveness of the proposed model in visual improvement as well as an increase in the peak signal-to-noise ratio comparing to corresponding PDE methods.

1. Introduction

Image de-noising is an inverse problem which has been widely studied in the areas of image processing and computer vision. The main purpose of de-noising is to remove unwanted components from image. These unwanted components are defined as noise and can be of different nature. Therefore, the process of approximating the unknown image of interest from the given noisy image known as image restoration or image de-noising, plays a key role in different fields. Applications that require a restoration operation range from astronomy, astrophysics, biology, chemistry, arts, geophysics, physics, hydrology, remote sensing and other areas involving imaging techniques [1].

In many image formulation models, noise is often modeling as an additive noise: given an original image u it is assumed to be corrupted by the additive noise η_1. The goal is then to recover u from the data $f = u+\eta_1$. There are many effective methods to tackle this problem. Among the most famous ones are wavelets approaches, stochastic approaches, principal component analysis-based approaches, and variational approaches, introduced by ROF [2]. There are two terms in the variational model; a regularizer and a data fidelity term. It becomes evident that variational approaches to the image de-noising problem have attracted much attention by directly approximating the reflectance of the underlying scene and yield often excellent results. Due to the edge preserving and noise removing properties, total variation approach has been widely utilized in the noise removal task. However, it has two main disadvantages

- In ROF model, the structure of image is modeled as a function belonging to the bounded variation (BV) space and therefore it favors a piecewise constant function in BV space which often causes the staircase effect.

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- The ROF method cannot preserve finer details such as textures well. According to [3], the l^2-norm cannot separate different oscillatory components with different frequencies such as textures and noise, and therefore the textures are filtered out with noise in the process of restoration. The interested reader is referred to [4–12] for more details.

In practice, there are other types of noise as well such as multiplicative noise. It can also degrade an image. In this paper, we deal with the multiplicative noise removal problem. Specifically, we are interested in removing the Gamma distributed multiplicative noise from a contaminated image through total variational approach. The assumption is that the original image u has been corrupted by some multiplicative noise η_2, the goal is then to recover u from the data $f = u\eta_2$ which follows a Gamma law with mean one and its probability density function is given by

$$G_{\eta_2}^{\upsilon} = \frac{L^L}{\Gamma(L)} \eta_2^{L-1} \exp(-L\eta_2) \tag{1}$$

where L is the number of looks (in general, an integer coefficient) and $\Gamma(\cdot)$ is a Gamma function.

Images propagated by coherent imaging systems, for example, ultrasound, synthetic aperture radar (SAR), and laser imaging, inevitably come with multiplicative noise (also known as speckle) due to the coherent nature of the scattering phenomena. The multiplicative noise seriously interferes with the upper tasks, such as image segmentation, object recognition or target detection and classification. Due to the coherent nature of the image acquisition process, in the multiplicative noise models, the noise field is multiplied by the original image and it is described by a non-Gaussian probability density functions, with Rayleigh and Gamma being common models [13,14]. It is signal independent, non-Gaussian and spatially dependent i.e. variance is a function of signal amplitude. Hence the removal of multiplicative noise is a very challenging task compared with additive Gaussian noise. Up to now, the popular despeckling methods include spatial, wavelet-based, non-local filtering and variational. In this study, we will focus on the variational approach for restoring images with multiplicative noise.

To the best of our knowledge, there exist several variational approaches devoted to multiplicative noise removal problem. We refer the reader to literature [9,15–28] and references included herein for an overview of the subject.

The total variational models have received considerable attention in signal and image processing community. Given an image $f \in L^2(\Omega)$, with $\Omega \subset R^2$ an open and bounded domain, the TV based models can be written uniformly as follows

$$\min_{u \in BV(\Omega)} \left\{ E(u) = J(u) + \lambda H(f, u) \right\} \tag{2}$$

where $J(u)$ is the total variation of u called regularized term, $H(f, u)$ is a data fidelity term and λ is the regularization parameter.

The classical algorithms for image de-noising do not properly approximate images containing edges. To overcome this, a technique based on the minimization of the total variation norm is proposed in [2]. This technique has been proved to be able to achieve a good tradeoff between edge preservation and noise removal. The images resulting from the application of this method in the presence of noise however tends to produce the so-called staircase effects on the images because it favors a piecewise constant solution in bounded variation space. Thus the image features in the original image may not be recovered satisfactorily and ramps will give piecewise constant regions (staircase effect). To reduce the blocky effects, while preserving sharp jump discontinuities, many methods have been proposed in the literature. The improved methods of total variation (TV) are divided into two kinds; the high order derivative and the fractional order derivative. For example, a fourth-order partial differential equations based noise removal model was proposed by [29]. It has been proved that this model is able to deal with the blocky problem. However, it causes the sign of uplifting effect and formation of artifacts around edges. To overcome this problem, an improved fourth-order PDE model was proposed in [30]. The interested researcher is referred to [3,9,15,31–33] for a more thorough discussion.

As a compromise between the first-order total variational regularized models and high-order derivative based models, some fractional-order derivative based models have been introduced in [33–37] for additive and multiplicative noise removal and subsequently used for image restoration and super-resolution [3,38]. They can ease the conflict between blocky-effect elimination and edge preservation by choosing the order of derivative properly. Moreover, the fractional-order derivative operator has a "non-local" behavior because the fractional-order derivative at a point depends upon the characteristics of the entire function and not just the values in the vicinity of the point [33], which is beneficial to improve the performance of texture preservation. The fractional-order total variation based model can be formulated as follows

$$\min_{u \in BV^{\alpha}(\Omega)} \left\{ E(u) = J^{\alpha}(u) + \lambda H(f, u),\ 1 \le \alpha \le 2 \right\} \tag{3}$$

where $J^{\alpha}(u) = \int_{\Omega} \sqrt{(\nabla_x^{\alpha} u)^2 + (\nabla_y^{\alpha} u)^2} dxdy$ is the fractional-order total variation of u and $\nabla_x^{\alpha} u$, $\nabla_y^{\alpha} u$ are the αth-order partial derivatives which are defined by

$$\nabla_x^{\alpha} u(x, y) = \lim_{\Delta x \to 0^+} \frac{\sum_{k \ge 0} (-1)^k C_{\alpha}^k u(x - k\Delta x, y)}{(\Delta x)^{\alpha}}; \qquad \nabla_y^{\alpha} u(x, y) = \lim_{\Delta y \to 0^+} \frac{\sum_{k \ge 0} (-1)^k C_{\alpha}^k u(x, y - k\Delta y)}{(\Delta y)^{\alpha}} \tag{4}$$