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Solitons and rouge waves for a generalized (3 + 1)-dimensional variable-coefficient Kadomtsev–Petviashvili equation in fluid mechanics

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ABSTRACT

Evolution of the long water waves and small-amplitude surface waves with the weak nonlinearity, weak dispersion and weak perturbation in fluid mechanics in three spatial dimensions can be described by a generalized (3 + 1)-dimensional variable-coefficient Kadomtsev–Petviashvili equation, which is studied in this paper with symbolic computation. Via the truncated Painlevé expansion, an auto-Bäcklund transformation is derived, based on which, under certain variable-coefficient constraints, one-soliton, two-soliton, homoclinic breather-wave and rouge-wave solutions are respectively obtained via the Hirota method. Graphic analysis shows that the soliton propagates with the varying soliton direction. Change of the value of any one of $g(t)$, $m(t)$, $n(t)$, $h(t)$, $q(t)$ and $l(t)$ in the equation can cause the change of the soliton shape, while the soliton amplitude cannot be affected by that change, where $g(t)$ represents the dispersion, $m(t)$ and $n(t)$ respectively stand for the disturbed wave velocities along the y and z directions, $h(t)$, $q(t)$ and $l(t)$ are the perturbed effects, y and z are the scaled spatial coordinates, and t is the temporal coordinate. Soliton direction and type of the interaction between the two solitons can vary with the change of the value of $g(t)$, while they cannot be affected by $m(t)$, $n(t)$, $h(t)$, $q(t)$ and $l(t)$. Homoclinic breather wave and rouge wave are respectively displayed, where the rouge wave comes from the extreme behaviour of the homoclinic breather wave.

1. Introduction

Nonlinear evolution equations (NLEEs) have been used to describe the nonlinear physical phenomena in such fields as fluid mechanics, plasma physics, optical fibres and solid state physics [1–9]. Among the NLEEs, Kadomtsev–Petviashvili (KP) hierarchy, containing the NLEEs like the Jimbo–Miwa equation and KP equation, has captured certain research attention [10]. The KP equation, a prototype of the two-dimensional NLEEs [11–13],

$$(U_T + 6UU_X + U_{XXX})_X + 3\sigma^2 U_{YY} = 0, \tag{1}$$

can describe the evolution of long water waves and small-amplitude surface waves with the weak nonlinearity, weak dispersion and weak perturbation in fluid mechanics, where U, a differentiable function of the longitudinal and transverse spatial coordinates X, Y and temporal coordinate T, represents the wave amplitude, the subscripts denote the partial

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derivatives, and $\sigma^2 = \pm 1$. Eq. (1) has been proved to be completely integrable [11]. Bäcklund transformation, bilinear forms and N-soliton solutions for Eq. (1) have been got [12,13].

Generalizations for Eq. (1) have been proposed [14–17]. Nowadays, those generalized KP equations have been investigated from quite a few perspectives, e.g., the Lax pair, Painlevé analysis, complete integrability, bilinear forms and analytic solutions [18–23]. In this paper, with symbolic computation [24–27], we plan to study a generalized $(3+1)$-dimensional variable-coefficient KP equation for the evolution of long water waves and small-amplitude surface waves with the weak nonlinearity, weak dispersion and weak perturbation in fluid mechanics in three spatial dimensions, written as [10,28]

$$\left[u_t + f(t)uu_x + g(t)u_{xxx} + h(t)u_x + q(t)u_y + l(t)u_z\right]_x + m(t)u_{yy} + n(t)u_{zz} = 0, \tag{2}$$

where u, a differentiable function of the scaled spatial coordinates x, y, z and temporal coordinate t, represents the wave amplitude, the nonzero differentiable functions $f(t)$ and $g(t)$ respectively represent the nonlinearity and dispersion, the differentiable functions $h(t)$, $q(t)$ and $l(t)$ are the perturbed effects, while the differentiable functions $m(t)$ and $n(t)$ stand for the disturbed wave velocities along the y and z directions, respectively [10,28]. Special cases for Eq. (2) have been seen in physical sciences, and have been studied, such as the (1+1)-dimensional variable-coefficient Korteweg–de Vries equation [29] and generalized variable-coefficient KP equation [30].

Investigation on the solitons and rouge waves for Eq. (2) via the Hirota method will be the main focus of this paper. In Section 2, we will derive the auto-Bäcklund transformation for Eq. (2) via the truncated Painlevé expansion. Through the Hirota method [31,32] with the obtained auto-Bäcklund transformation, we will study the solitons and rouge waves respectively in Sections 3 and 4. In Section 3, we will give the one- and two-soliton solutions, based on which the evolution of the solitons as well as the effects of the variable coefficients in Eq. (2) on the solitons will be discussed; In Section 4, homoclinic breather-wave solutions and rouge-wave solutions will be got, where the rouge-wave solutions come from the extreme behaviour of the homoclinic breather-wave solutions. Section 5 will be the conclusions.

2. Auto-Bäcklund transformation for Eq. (2)

Auto-Bäcklund transformation is a relation between a solution to a differential equation and another solution to the same differential equation, which provides a means of constructing the new solutions from the known ones [33]. Hereby, truncated Painlevé expansion [34] will be applied to derive the auto-Bäcklund transformation for Eq. (2).

Now, we study the Painlevé expansion in a generalized Laurent series truncated at the constant-level term, i.e.,

$$u(x,y,z,t) = \phi(x,y,z,t)^{-K}\sum_{j=0}^{K} u_j(x,y,z,t)\phi(x,y,z,t)^j, \tag{3}$$

where K is the natural number, u_j's and ϕ are the analytic functions with $u_0 \neq 0$ and $\phi_x \neq 0$. The leading-order analysis [12] gives

$$K = 2. \tag{4}$$

Substituting Expressions (3) and (4) into Eq. (2) and making the coefficients of like powers of ϕ to vanish, with symbolic computation, we get the following Painlevé–Bäcklund equations:

$$\phi^{-6}: \quad u_0 = -\frac{12g(t)\phi_x^2}{f(t)}, \tag{5a}$$

$$\phi^{-5}: \quad u_1 = \frac{12g(t)\phi_{xx}}{f(t)}, \tag{5b}$$

$$\begin{aligned}\phi^{-4}: \quad & g(t)n(t)\phi_t^2\phi_x^2 + g(t)m(t)\phi_y^2\phi_x^2 + g(t)\phi_t\phi_x^3 + g(t)l(t)\phi_z\phi_x^3 + g(t)q(t)\phi_y\phi_x^3 \\ & + g(t)h(t)\phi_x^4 + g(t)f(t)u_2\phi_x^4 - 3g(t)^2\phi_x^2\phi_{xx}^2 + 4g(t)^2\phi_x^3\phi_{xxx} = 0,\end{aligned} \tag{5c}$$

$$\begin{aligned}\phi^{-3}: \quad & f(t)g(t)n(t)\phi_{zz}\phi_x^2 + f(t)g(t)m(t)\phi_{yy}\phi_x^2 + g(t)f'(t)\phi_x^3 + g'(t)f(t)\phi_x^3 \\ & + 2g(t)f(t)^2\phi_x^3u_{2,x} + 3g(t)f(t)\phi_x^2\phi_{xt} + 4g(t)f(t)n(t)\phi_z\phi_x\phi_{xz} \\ & + 3g(t)f(t)l(t)\phi_x^2\phi_{xz} + 4g(t)f(t)m(t)\phi_y\phi_x\phi_{xy} + 3g(t)f(t)q(t)\phi_x^2\phi_{xy} \\ & + g(t)n(t)f(t)\phi_z^2\phi_{xx} + g(t)m(t)f(t)\phi_y^2\phi_{xx} + 3g(t)f(t)\phi_t\phi_x\phi_{xx} \\ & + 3g(t)f(t)l(t)\phi_z\phi_x\phi_{xx} + 3g(t)f(t)q(t)\phi_y\phi_x\phi_{xx} + 6g(t)f(t)h(t)\phi_x^2\phi_{xx} \\ & + 6g(t)f(t)^2u_2\phi_x^2\phi_{xx} - 3g(t)^2f(t)\phi_{xx}^3 + 9g(t)^2f(t)\phi_x^2\phi_{xxxx} = 0,\end{aligned} \tag{5d}$$

$$\begin{aligned}\phi^{-2}: \quad & 2g(t)f(t)n(t)\phi_{xz}^2 + 2g(t)f(t)n(t)\phi_x\phi_{xzz} + 2g(t)f(t)m(t)\phi_{xy}^2 \\ & + 2g(t)f(t)m(t)\phi_x\phi_{xyy} + g(t)f(t)n(t)\phi_{zz}\phi_{xx} + g(t)m(t)f(t)\phi_{xx}\phi_{yy} \\ & - 3g(t)f'(t)\phi_x\phi_{xx} + 3g'(t)f(t)\phi_x\phi_{xx} + 6g(t)f(t)^2\phi_xu_{2,x}\phi_{xx}\end{aligned}$$