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Notes on "The Cattaneo-type time fractional heat conduction equation for laser heating" [Comput. Math. Appl. 66 (2013) 824–831]

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ABSTRACT

In this paper, Eq. (6) in Qi et al. (2013) is corrected by pointing out the missing time delayed fractional derivative item of $I_1\delta \cdot g\left(x\right) \tau^p D_t^p f\left(t\right)/k$. The time fractional heat conduction model is used as the constitutive heat diffusion model and the corresponding fractional heat conduction equation with a volumetric heat source is built. Subjected to the same initial and boundary conditions as those in Qi et al. (2013), the analytical solution is presented. Furthermore, the dimensionless temperature variations and distributions are shown graphically with various values of fractional order and delayed time parameters. Compared with the previous curves in Qi et al. (2013), the obtained temperature variation and distribution are different and even opposite.

1. Introduction

For Eq. (6) in Ref. [1], the mathematical model on Cattaneo-type fractional heat conduction for laser heating is given as

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} + \frac{\tau^p}{\alpha} D_t^{1+p} T = \frac{\partial^2 T}{\partial x^2} + \frac{I_1 \delta}{k} f\left(t\right) g\left(x\right) \tag{1}$$

where T is temperature, α thermal diffusivity, K thermal conductivity, τ the delayed time, t time, x space coordinate. $I_1 \left(t = \left(1 - r_f\right) I_0\right)$ (r_f is the reflection coefficient, I_0 is the peak power intensity of the laser pulses) is surface absorption intensity. $D_t^{1+p} T (p \in [0, 1])$ is the Caputo fractional derivative of $(1 + p)$ order, $f\left(t\right)$ is the temporal function representing the laser pulse intensity variation, and $g\left(x\right)$ is the absorption term.

It is noted that the above equation is not correct for Cattaneo-type fractional heat conduction model with laser heating source. Therefore, the analytical solution in Ref. [1] is not available to the problems for laser heating. The corrected Cattaneo-type fractional heat conduction model with laser heating source and the corresponding solution with the same initial and boundary condition as those in Ref. [1] are given in the following sections.

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2. Mathematical model and analysis of fractional heat conduction

2.1. General fractional Cattaneo model

The modified constitutive relationship between the flux and temperature gradient by Cattaneo [2] and Vernotte [3] is

$$q+\tau\frac{\partial q}{\partial t}=-k\nabla T. \tag{2}$$

Replacing the integer order derivative to time with the fractional order derivative, Eq. (2) becomes

$$q+\tau^{p}D_t^{p}q=-k\nabla T \quad 0<p<1. \tag{3}$$

The general energy conservation law is taken as

$$\rho c_p\frac{\partial T}{\partial t}=-\nabla\cdot q+Q \tag{4}$$

where ρ is the material density, c_P is the material specific thermal capacity, and Q is the volumetric heat source.

For the laser heating, Q is taken as the same model as that in Ref. [1]

$$Q=I_1\delta\cdot f\left(t\right)g\left(x\right). \tag{5}$$

Combining Eq. (3) with Eqs. (4)–(5), the general fractional heat conduction equation is drawn out

$$\frac{1}{\alpha}\frac{\partial T}{\partial t}+\frac{\tau^{p}}{\alpha}D_t^{p+1}T=\nabla^2T+\frac{I_1\delta\cdot g\left(x\right)}{k}\left(1+\tau^{p}D_t^{p}\right)f\left(t\right) \quad 0<p<1. \tag{6}$$

When $p=1$, Eq. (6) becomes the hyperbolic heat conduction with internal heat source [4]. It can be noted that ∇T of Eq. (5) in Ref. [1] should be $\nabla^2 T$.

Considering ultra-short time pulse laser heating, one dimension mathematical model of Cattaneo fractional heat conduction with laser heating source is given

$$\frac{1}{\alpha}\frac{\partial T}{\partial t}+\frac{\tau^{p}}{\alpha}D_t^{p+1}T=\frac{\partial^2T}{\partial x^2}+\frac{I_1\delta\cdot g\left(x\right)}{k}\left(1+\tau^{p}D_t^{p}\right)f\left(t\right) \quad 0<p<1. \tag{7}$$

It can be found that the laser heating provides the heat source, which depends on t, x, and p. Compared with Eq. (6) in Ref. [1], Eq. (7) here preserves the fractional order time delayed item of laser heating source $I_1\delta\cdot g\left(x\right)\tau^{p}D_t^{p}f\left(t\right)/k$. That is to say, Eq. (6) in Ref. [1] misses the item.

Considering the same initial and boundary conditions as those in Ref. [1], the time variation function of laser and space distribution function of the laser intensity is also the same as those in Ref. [1]

$$f\left(t\right)=H\left(t\right)-H\left(t-t_d\right), \qquad g\left(x\right)=e^{-\delta x} \tag{8}$$

where $H\left(\cdot\right)$ is the Heaviside unit step function, and t_d is the pulse duration of the step pulse.

2.2. Solution of temperature distribution

Adopting the same dimensionless quantities as those in Ref. [1], dimensionless equation groups are given as

$$\frac{\partial T}{\partial t}+\tau^{p}\frac{\partial^{p+1}T}{\partial t^{p+1}}=\frac{\partial^2T}{\partial x^2}+\left(1+\tau^{p}\frac{\partial^{p}}{\partial t^{p}}\right)\exp\left(-x\right)f\left(t\right) \tag{9}$$

$$T\left(x,t\right)=0, \qquad \frac{\partial}{\partial t}T\left(x,t\right)=0 \quad x>0,\ t=0 \tag{10}$$

$$\frac{\partial T}{\partial x}=0 \quad x=0,\ t>0 \tag{11}$$

$$T\left(x,t\right)=0 \quad x\to\infty,\ t=0 \tag{12}$$

where $f\left(t\right)=H\left(t\right)-H\left(t-t_d\right)$.

The Laplace transform solution of above equation groups in the transformed domain is

$$\overline{T}\left(x,s\right)=\overline{T_1}\left(x,s\right)-\exp\left(-t_ds\right)\overline{T}_1\left(x,s\right) \tag{13}$$