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A feasible QP-free algorithm combining the interior-point method with active set for constrained optimization☆

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ABSTRACT

In this paper, by means of "working set" technique for determining the active set and the idea of primal–dual interior-point method, a new feasible QP-free algorithm for solving inequality constrained optimization problems is presented. At each iteration, the algorithm solves only three reduced systems of linear equations with common coefficient matrix. Moreover, the initial iteration point can be at constraint boundary and the coefficient matrix is uniformly nonsingular without the strict complementarity. We also prove that the proposed algorithm obtains global and superlinear convergence. At last, preliminary numerical results are reported.

1. Introduction

In this paper, we consider the following problem

$$(\mathrm{P}) \qquad \begin{array}{ll} \min & f(x) \\ \text{s.t.} & g_i(x) \le 0, \quad i \in I = \{1, 2, \ldots, m\}. \end{array} \tag{1.1}$$

Denote the feasible set for problem (P) by $X = \{x \in R^n : g_i(x) \le 0,\ i \in I\}$. A given point x is said to be a KKT point for (P) if there is $\lambda \in R^m$ such that

$$\begin{cases} \nabla_x L(x, \lambda) = 0 \\ \lambda_i g_i(x) = 0, \quad \lambda_i \ge 0,\ g_i(x) \le 0,\ i \in I, \end{cases} \tag{1.2}$$

where $L(x, \lambda) = f(x) + \sum_{i \in I} \lambda_i g_i(x)$ is the Lagrangian function of (P). And a point x is called a stationary point for (P) if (x, λ) satisfies (1.2) except for $\lambda_i \ge 0,\ i \in I$.

Sequential quadratic programming (SQP) methods are a class of efficient methods for solving nonlinearly constrained optimization problems. For a further survey on SQP methods, one can refer to Refs. [3,4].

However, SQP methods requiring solving QP subproblems at every iteration spend a lot of care on computation. In [5], Panier, Tits and Herskovits proposed a feasible QP-free algorithm. At each iteration, only three systems of linear equations

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need to be solved. The process of their QP-free algorithm is as follows. To yield the search direction, they first solve two systems of linear equations of the form

$$\begin{pmatrix} H_k & A_k \\ Z_k A_k^{\mathrm{T}} & G_k \end{pmatrix} \begin{pmatrix} d \\ \lambda \end{pmatrix} = \begin{pmatrix} -\nabla f(x^k) \\ \mu \end{pmatrix}, \tag{1.3}$$

by choosing a different vector μ. Where $Z_k = \mathrm{diag}(z^k)$, a diagonal matrix whose ith diagonal element is z_i^k, the ith element of vector $z^k \in R^m$, $G_k = \mathrm{diag}(g_i(x^k), i \in I)$, $A_k = (\nabla g_i(x^k),\ i \in I)$. To avoid the so-called Maratos effect, they further solve a least squares subproblem, which is equivalent to a system of linear equations. However, this QP-free algorithm may be unstable, since system (1.3) may become very ill-conditioned if some multiplier z_i^k corresponding to a nearly active constraint g_i becomes very small. Further, to get global convergence, an additional assumption that the number of stationary points is finite is used. The algorithm was later improved by Gao et al. [6] to overcome this shortcoming by solving an extra system of linear equations. But they assume that the multiplier approximation sequence remains bounded. Later, by means of the Fischer–Burmeister function, Qi and Qi [7] proposed a new feasible QP-free algorithm for solving (P). At each iteration, it is required to solve three systems of linear equations of the form

$$\begin{pmatrix} H_k & \nabla g(x^k) \\ \mathrm{diag}(\eta^k)\nabla g(x^k)^{\mathrm{T}} & -\sqrt{2}\mathrm{diag}(\theta^k) \end{pmatrix} \begin{pmatrix} d \\ \lambda \end{pmatrix} = \begin{pmatrix} -\nabla f(x^k) \\ c \end{pmatrix}, \tag{1.4}$$

where c is a suitable vector and for each $i \in I$

$$\eta_i^k := \frac{g_i(x^k)}{\sqrt{g_i^2(x^k) + (\mu_i^k)^2}} + 1, \qquad \theta_i^k := \left(1 - \frac{\mu_i^k}{\sqrt{g_i^2(x^k) + (\mu_i^k)^2}}\right)^{\frac{1}{2}}.$$

The coefficient matrix of (1.4) is nonsingular even if the strict complementarity does not hold. The method achieves global convergence without requiring the isolatedness of the stationary points.

Primal–dual interior-point methods [8,9] have enjoyed increasing popularity in recent years. The basis of the primal–dual feasible interior-point iteration is the system of equations of (1.2) in (x, λ) with a perturbation as follows

$$\begin{cases} \nabla_x L(x, \lambda) = 0 \\ \lambda_i g_i(x) = \eta_i, \qquad i \in I. \end{cases} \tag{1.5}$$

The vector η is the "barrier parameter" vector with element $\eta_i < 0,\ i \in I$. The idea is then to attempt to solve (1.5) by means of Newton or quasi-Newton iteration, while driving η to zero and enforcing primal and dual (strict) feasible at each iteration. Specially, the following linear system in $(\Delta x, \Delta\lambda)$ is considered at pair (x^k, λ^k)

$$\begin{aligned} &H_k \Delta x + \sum_{i \in I} \nabla g_i(x^k) \Delta\lambda_i + \nabla f(x^k) + \sum_{i \in I} \nabla g_i(x^k) \lambda_i^k = 0, \\ &\lambda_i^k \nabla g_i(x^k)^{\mathrm{T}} \Delta x + g_i(x^k) \Delta\lambda_i + \lambda_i^k g_i(x^k) = \eta_i, \quad i \in I, \end{aligned}$$

or equivalently

$$\begin{pmatrix} H_k & A_k \\ \mathrm{diag}(\lambda^k) A_k^{\mathrm{T}} & G_k \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta\lambda + \lambda^k \end{pmatrix} = \begin{pmatrix} -\nabla f(x^k) \\ \eta \end{pmatrix}, \tag{1.6}$$

which is the same as system (1.3) except that each component of η is negative, where H_k equals or approximates the Hessian of Lagrangian with respect to x.

Using the idea of primal–dual interior-point method mentioned above, Bakhtiari and Tits proposed a new QP-free algorithm [1] to overcome the weaknesses of [5] discussed above. In [1], they only solved two systems of linear equations like (1.6) with different η to get search direction and solved a least squares subproblem to compute correction direction. Particularly, in the second system, they used the idea of interior-point methods to construct the barrier parameter vector $\eta < 0$ carefully. Without the additional assumption of isolatedness of the stationary points, the algorithm achieves global convergence. Also, another useful improvement of this algorithm is that it can start from a feasible point at boundary of the feasible set. But they used all the constraints and their gradients to construct the coefficient matrix. As a result, the scale of the system of linear equations may be very large. Moreover, the coefficient matrix of the third system is not the same as the previous ones, which adds the computation cost to some extent.

In this paper, we try to overcome the weaknesses in [1] mentioned in the paragraph above. First, we use the working set I_k [10] as an estimate of the active set $I(x^k)$, where $I(x^k) = \{i \in I :\ g_i(x^k) = 0\}$. And I_k is an identification of the active set $I(x^*)$ when x^k is sufficiently close to a KKT point x^* for (P). The working set and the identification of the active set have been studied by some authors [10,11]. With the help of working set I_k, at each iteration, we solve only three reduced systems of linear equations with same coefficient matrix as follows

$$\begin{pmatrix} H_k & A_{I_k} \\ Z_{I_k} A_{I_k}^{\mathrm{T}} & G_{I_k} \end{pmatrix} \begin{pmatrix} d \\ \lambda_{I_k} \end{pmatrix} = \begin{pmatrix} -\nabla f(x^k) \\ \mu \end{pmatrix}, \tag{1.7}$$