Computers and Mathematics with Applications 70 (2015) 2803–2821

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Wave propagation in a rotating randomly varying granular generalized thermoelastic medium

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ARTICLE INFO

Article history:
Received 22 January 2015
Received in revised form 1 July 2015
Accepted 12 September 2015
Available online 27 October 2015

Keywords:
Wave propagation
Rotating
Random
Granular
Generalized thermo-elasticity

ABSTRACT

Elastic wave propagation has been explored in an infinite granular thermoelastic medium rotating with constant speed. The elastic and thermal parameters of the granular medium are taken to be randomly fluctuated so that the medium represents the randomly fluctuating inhomogeneous medium. The method of smooth perturbation has been used, which requires the inversion of a deterministic differential operator to find the solution of governing equations in the relevant media. The analysis is based on the dynamics of granular medium as propounded by N. Oshima. All field parameters are functions of space vector and time. A general dispersion equation for waves propagating in the rotating random granular generalized thermal elastic medium has been obtained. The compression and shear wave propagations have been studied. It has been pointed out that in the case of compression waves, the mean and auto-correlation function of the thermo-mechanical coupling parameter greatly influence the mean wave propagation. For shear waves, however, randomness has no effect on wave propagation. Effects of non-random granular elastic medium, randomness and rotation of the frame of reference are discernible from analyses of dispersion equations. The study may find applications in soil mechanics, seismology and oil-prospecting. Computational results have been shown.

1. Introduction

The study of randomness in relation to wave propagation in different media has been steadily drawing attention of applied mathematicians from the time of Chernov [1]. Various aspects of wave propagation phenomena are being investigated by authors with a view to characterizing random variation of properties of medium. The propagation of elastic waves, among others, in a random medium was treated by Keller [2] by using the method of smooth perturbation. Keller's smooth perturbation technique requires an inversion (L_0^{-1}) of the deterministic differential operator L_0 to be performed in order to enable the evaluation of the mean field quantity. Accordingly, in every application of this technique, the computation of an appropriate Green's tensor becomes unavoidable. Ryzhik, Papanicolaou and Keller [3] derived and analysed transport equations for energy density of waves of any kind in random medium. Beran and McCoy [4] enunciated iterative perturbation theory in discussing mean field variation in random dielectric and other media. The mean field equation, unlike Keller's smooth perturbation technique, was presented in terms of an infinite sequence of correlation functions associated with the field parameter(s). In another paper Beran and McCoy [5] discussed in detail the method of determination of the mean field quantities in a statistical sample of heterogeneous linearly elastic solids. Beran, Frankenthal, Deshmukh and Whitman [6] studied propagation of radiation in time-dependent three-dimensional random media. Many other methods pertaining to

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http://dx.doi.org/10.1016/j.camwa.2015.09.015
0898-1221/

Nomenclature

$\vec{u}$	Linear displacement vector
$\vec{\xi}$	Rotation vector
θ	Generalized temperature
λ	Lame' parameter
μ	Lame' parameter
A	Coefficient of friction between the individual grains
B	Third elastic parameter
ν	Thermal diffusivity
ρ	Density
α	Thermal expansion coefficient
C_s	Specific heat
m	Thermomechanical coupling parameter
t_0, t_1	Generalized thermal parameters
θ_0	Reference temperature
$\vec{F}$	Constant linear body force
$\vec{\psi}$	Angular body force
$\vec{Q}$	Heat source
$\vec{k}$	Wave vector
δ_{lk}	Kronecker delta.

(Graphs were drawn taking $\delta_{lk} = 1 \cdot 0$ *conforming to L–S theory*).

random character of inhomogeneities of media have been developed. A detailed mathematical discussion can be found in treatises of Bharucha Reid [7], Uscinski [8] and others. Sobczyk [9a] studied elastic wave propagation in a discrete random medium. He developed a general formalism of the analysis of mean (or coherent) elastic waves in terms of scatterers. Sobczyk, Wedrychowicz and Spencer Jr [9b] analysed the dynamics of structural systems with randomly varying parameters. Wenzel [10] made a theoretical study on radiation and attenuation of waves in a random medium. Chow [11] employed smooth perturbation in studying thermoelastic waves in a random medium. Bera [12] investigated wave propagation in a rotating magneto-thermoelastic medium under generalized thermoelasticity involving two relaxation times, t_0, t_1, studying randomness of interacting media employing smooth perturbation method. Bhattacharyya [13a] investigated the problem of reflection of waves, incident on the plane boundary of a semi-infinite elastic medium with randomly varying inhomogeneities. Mitra and Bhattacharyya [13b] recently studied wave propagation in a random micropolar thermal elastic medium, second moments and associated Green's tensor.

In this paper we shall adopt Keller's method to analyse the mean generalized thermo-elastic wave in a slightly random, inhomogeneous granular elastic medium under a rotating frame of reference. By this we mean that the density, the *Lame*' constants, the thermal expansion coefficient, the specific heat, the thermal diffusivity and parameters defining granular character of the elastic medium are all random functions of positions. The medium is assumed to rotate under a frame of reference defined in Schöenberg and Censor [14]. Many authors have studied effect of rotation for different types of problems of wave propagation in different media. Recently Tomar and Ogden [15] studied wave propagation in an isothermal linear isotropic elastic material with voids rotating with constant angular velocity about the z-axis. On the other hand, Bakshi, Bera and Debnath [16] investigated effects of rotation in studying magneto-thermoelastic problems with thermal relaxation and heat sources in a three-dimensional infinite elastic medium. Earlier, Roy Chaudhuri and Debnath [17] investigated magneto-thermoelastic plane waves in rotating media; effects of rotation, though small, were computed for the phase speed, attenuation and specific energy loss, etc.

The granular character of the medium has been defined under the dynamics of granular medium described by Oshima [18] as a Cosserat-continuum. The study of the granular character of the medium finds importance in soil mechanics, earthquake source mechanism in the crustal layer of the earth and in petroleum engineering. The effects though small of the order of second order perturbation, nonetheless, come to influence computational results. The medium under consideration is a discontinuous one being composed of numerous large and small grains. Oshima visualizes that 'unlike a continuous body each element or grain not only translates but also rotates about its centre of gravity. The later motion affects the equation of motion by producing internal friction. It is assumed that the medium contains so many grains that they will never be separated from each other during the deformation and the grain has perfect elasticity.' He uses the concept of averages to reduce the granular medium to a continuous medium by introducing additional physical constants [19]. The stress tensor is not symmetrical. Besides the non-symmetrical stress-tensor there is also the non-symmetric stress couple which is related to a non-symmetric strain-tensor. Also the couple stress theory developed by Eringen [20,21] comprises granular as also composite fibrous materials. As such micropolar or microstructure materials and granular materials present an inclusive model of composite materials.