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Two-scale analysis for environmental dispersion in a two-layer wetland

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ABSTRACT

Studies on environmental dispersion are essential for applications as water management. The two-scale perturbation analysis is applied in this paper to deduce the environmental dispersion model for the typical case of contaminant transport in two-layer wetland flows. The analysis follows the established theoretical framework on the basis of phase average and the concept of Taylor dispersion. By the obtained flow velocity distribution for the two-layer flow, the analytical expression for the environmental dispersivity is deduced and shown to be consistent with previous results by the concentration moment method, while with much simplifications on the expression for ignoring the less concerned time-dependent stage of the dispersivity.

1. Introduction

Contaminant transport in wetland flows is directly involved in water supply, water quality control, and risk assessment and management (Chen et al., 2011;Shao and Chen, 2013; Shao et al., 2013b). Understanding the transport processes, which provides information on the fate of soluble pollutants, forms the scientific basis for the associated applications (Chen et al., 2010;Lightbody and Nepf, 2006; Zeng et al., 2012; Wang and Chen, 2015). Exactly due to the essential implications, intensive studies have been carried out in recent years (Nepf, 2012;Shao et al., 2013a; Wang et al., 2013;Wu et al., 2011c).

Among the many researches, the theoretical framework established by Professor G.Q. Chen and his collaborators (Wu and Chen, 2012;Wu et al., 2011a; Wu et al., 2012;Zeng, 2010) provides a useful one-dimensional environmental dispersion model, which was based on the phase-average theory and the concept of Taylor dispersion (Taylor, 1953). By phase average the discontinuities of both velocity and concentration in space caused by the vegetation in wetlands are removed, and by Taylor dispersion concept the transport of the contaminant can be described by an effective diffusion equation under long-time evolution (Wu and Chen, 2014b;Zeng, 2010).

Under the framework, the most idealized case of contaminant transport in a homogeneous wetland channel has been studied (Chen et al., 2010). Wu et al. (2011a) extended the analysis to consider a two-zone wetland channel, as a start point of studying environmental dispersion in wetland flows of a multi-zone structure. To analyze the effects of ecological degradation on the transport process, Chen (2013) included a first order reaction model in the basic concentration transport equation in deducing the analytical solution of environmental dispersivity for the two-zone wetland flow, as an extension on the work of Wu et al. (2011a).

For determining environmental dispersivities for contaminant transport in wetlands, it is Wu et al. (2011b) first adopted the multi-scale perturbation analysis. Also known as the homogenization technique (Mei et al., 1996) and as recently extended by Wu and Chen (2014b) for presenting the physical insights of Taylor dispersion, the method is very illustrative in dealing with complicated problems of multiple scales. In the analysis of Wu et al. (2011b) three scales are chosen, corresponding to the greater diffusion time-scale with molecular diffusion across a large longitudinal distance, the convection time-scale, and the smaller diffusion time-scale with molecular diffusion across the width of the wetland channel. By expanding the concentration into multiple scales, the complex processes are decoupled at different scales, and the effective diffusion equation can be obtained by successively considering the perturbation problems to the second order.

By now the multi-scale analysis has not been applied to study contaminant transport in a two-layer wetland. It has been shown that by the multi-scale analysis (Wu et al., 2011b), the expression for the obtained analytical solution of the environmental dispersivity is much simpler than that obtained by the method

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of concentration moment (Aris, 1956;Wu et al., 2011a). This is because only the concerned asymptotic evolution of the contaminant transport is analyzed by the method. Additionally for describing the long-time evolution of the contaminant cloud, two time scales are enough instead of three (Chen and Wu, 2012). In this paper, environmental dispersion for the typical type of two-layer wetland flow with a free water surface is to be considered for analytical solution by the two-scale analysis.

2. Formulation for concentration transport

According to Taylor's classical experiment and related researches (Chen et al., 2012; Taylor, 1953; Wu and Chen, 2014a,b), there exist two stages after an instantaneous release of contaminant into the wetland flow. Within the initial stage of the contaminant transport, which is characterized by the transverse-mixing time-scale H^2/D^*, where H is the depth of the channel and D^* is the effective molecular diffusivity, the transverse mean concentration forms a skewed longitudinal distribution. In the latter stage, the vertical concentration difference decreases to a small fraction of its initial value, and the mean concentration tends to be in a Gaussian distribution, which is known as Taylor dispersion of the transport (Wu and Chen, 2014b;Zeng, 2010).

General equation for concentration transport can be adopted at the phase average scale as (Liu and Masliyah, 2005)

$$\phi\frac{\partial C}{\partial t}+\nabla\cdot(\mathbf{U}C)=\nabla\cdot(\kappa\lambda\phi\nabla C)+\kappa\nabla\cdot(\mathbf{K}\cdot\nabla C), \tag{1}$$

where $\mathbf{U}$ is velocity, t time, ϕ porosity, κ tortuosity, C concentration, λ concentration diffusivity, and $\mathbf{K}$ concentration dispersivity tensor.

The two-layer wetland is of height (H_1+H_2), or H, with H_1 and H_2 standing for the height of layer 1 and layer 2, respectively. In the Cartesian coordinate system, x-axis is aligned with the flow direction; z is the vertical coordinate with z_1 and z_2 respectively for layer 1 and 2; and origin is set at the wetland bed wall, as shown in Fig. 1.

For contaminant transport in the depth-dominated two-layer wetland flows, Eq. (1) is reduced to

$$\frac{\partial C_1}{\partial t}+\frac{u_1}{\phi_1}\frac{\partial C_1}{\partial x}=\kappa_1\left(\lambda_1+\frac{K_1}{\phi_1}\right)\frac{\partial^2 C_1}{\partial x^2}+\kappa_1\left(\lambda_1+\frac{K_1}{\phi_1}\right)\frac{\partial^2 C_1}{\partial z_1^2}, \tag{2}$$

$$\frac{\partial C_2}{\partial t}+\frac{u_2}{\phi_2}\frac{\partial C_2}{\partial x}=\kappa_2\left(\lambda_2+\frac{K_2}{\phi_2}\right)\frac{\partial^2 C_2}{\partial x^2}+\kappa_2\left(\lambda_2+\frac{K_2}{\phi_2}\right)\frac{\partial^2 C_2}{\partial z_2^2}, \tag{3}$$

where u_1 and u_2 are velocities, C_1 and C_2 the contaminant concentrations, κ_1 and κ_2 the tortuosities, λ_1 and λ_2 the concentration diffusivities, ϕ_1 and ϕ_2 the porosities, as well as K_1 and K_2 the vertical concentration dispersivities in layer 1 and layer 2, respectively.

Considering a uniform and instantaneous release of contaminant with mass Q at the cross-section of $x=0$ at time $t=0$, the initial conditions can be set as

$$C_1(x,z_1,t)|_{t=0}=\frac{Q\delta(x)}{\phi_1(H_1+H_2)}, \tag{4}$$

$$C_2(x,z_2,t)|_{t=0}=\frac{Q\delta(x)}{\phi_2(H_1+H_2)}, \tag{5}$$

where $\delta(x)$ is the Dirac delta function.

Since the amount of released contaminant is finite, boundary conditions for concentration at $x=\pm\infty$ can be written as

$$C_1(x,z_1,t)|_{x=\pm\infty}=C_2(x,z_2,t)|_{x=\pm\infty}=0. \tag{6}$$

Boundary conditions at the wetland bed of $z_1=0$ and the free-water-surface of $z_2=H_1+H_2$ read as

$$\left.\frac{\partial C_1(x,z_1,t)}{\partial z_1}\right|_{z_1=0}=\left.\frac{\partial C_2(x,z_2,t)}{\partial z_2}\right|_{z_2=H_1+H_2}=0. \tag{7}$$

3. Multi-scale analysis for the concentration transport

For contaminant dispersion in the two-layer wetland, the relation between $T_1=L/\langle u\rangle$ as the convection time scale, and $T_2=L^2\Big/\left\langle\kappa\left(\lambda+\frac{K}{\phi}\right)\right\rangle$ as the diffusion time scale can be generally expressed as

$$T_1:T_2=1:\frac{1}{\varepsilon}, \tag{8}$$

where $\varepsilon=H/L\ll 1$ and L is a characteristic length of the contaminant cloud.

With dimensionless parameters of

$$\xi=\frac{x}{L},\quad \psi_1=\frac{u_1}{u},\quad \psi_2=\frac{u_2}{u},\quad \tau=\frac{t}{L/u},\quad \zeta_1=\frac{z_1}{H},\quad \zeta_2=\frac{z_2}{H} \tag{9}$$

the governing equations for concentration transport and corresponding boundary and initial conditions can be rewritten as

$$\varepsilon\cdot\frac{\partial C_1}{\partial\tau}+\psi_{\phi1}\cdot\varepsilon\cdot\frac{\partial C_1}{\partial\xi}=\frac{1}{Pe_1}\cdot\varepsilon^2\cdot\frac{\partial^2 C_1}{\partial\xi^2}+\frac{1}{Pe_1}\cdot\frac{\partial^2 C_1}{\partial\zeta_1^2}, \tag{10}$$

$$\varepsilon\cdot\frac{\partial C_2}{\partial\tau}+\psi_{\phi2}\cdot\varepsilon\cdot\frac{\partial C_2}{\partial\xi}=\frac{1}{Pe_2}\cdot\varepsilon^2\cdot\frac{\partial^2 C_2}{\partial\xi^2}+\frac{1}{Pe_2}\cdot\frac{\partial^2 C_2}{\partial\zeta_2^2}, \tag{11}$$

$$\left.\frac{\partial C_1}{\partial\zeta_1}\right|_{\zeta_1=0}=0,\quad \left.\frac{\partial C_2}{\partial\zeta_2}\right|_{\zeta_2=1}=0, \tag{12}$$

where

$$\psi_{\phi1}=\frac{\psi_1}{\phi_1},\quad \psi_{\phi2}=\frac{\psi_2}{\phi_2}, \tag{13}$$

and

$$Pe_1=\frac{\langle u\rangle H}{\kappa_1\left(\lambda_1+\frac{K_1}{\phi_1}\right)},\quad Pe_2=\frac{\langle u\rangle H}{\kappa_2\left(\lambda_2+\frac{K_2}{\phi_2}\right)} \tag{14}$$

are Péclet numbers. The angle-brackets stand for the operation of depth-average defined for a variable f as

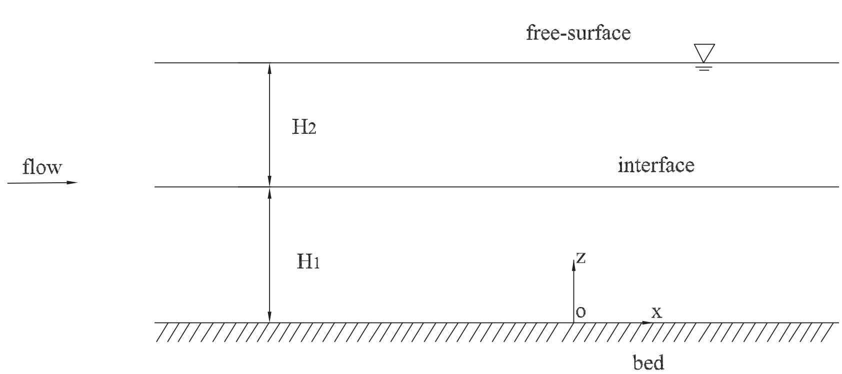

Fig. 1. Sketch for a two-layer wetland flow.