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Evidence of non extensivity in the evolution of seismicity along the San Andreas Fault, California, USA: An approach based on Tsallis statistical physics

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ABSTRACT

We examine the nature of the seismogenetic system along the San Andreas Fault (SAF), California, USA, by searching for evidence of complexity and non-extensivity in the earthquake record. We use accurate, complete and homogeneous earthquake catalogues in which aftershocks are included (raw catalogues), or have been removed by a stochastic declustering procedure (declustered catalogues). On the basis of Non-Extensive Statistical Physics (NESP), which generalizes the Boltzmann–Gibbs formalism to non-equilibrating (complex) systems, we investigate whether earthquakes are generated by an extensive self-excited Poisson process or by a non-extensive complex process. We examine bivariate cumulative frequency distributions of earthquake magnitudes and interevent times and determine the size and time dependence of the respective magnitude and temporal entropic indices, which indicate the level on non-equilibrium (correlation). It is shown that the magnitude entropic index is very stable and corresponds to proxy *b*-values that are *remarkably* consistent with the *b*-values computed by conventional means. The temporal entropic index computed from the raw catalogues indicate moderately to highly correlated states during the aftershock sequences of large earthquakes, progressing to quasi-uncorrelated states as these die out and before the next large event. Conversely, the analysis of the declustered catalogues shows that background seismicity exhibits moderate to high correlation that varies significantly albeit *smoothly* with time. This indicates a *persistent* sub-extensive seismogenetic system. The degree of correlation is generally higher in the southern SAF segment, which is consistent with the observation of shorter return periods for large earthquakes. A plausible explanation is that because aftershock sequences are localized in space and time, their efficient removal unveils long-range background interactions which are obscured by their presence! Our results indicate complexity in the expression of background seismicity along the San Andreas Fault, with criticality being a very likely mechanism as a consequence of the persistent non-equilibrium inferred from the temporal entropic index. However, definite conclusions cannot be drawn until the earthquake record is exhaustively studied in all its forms.

1. Introduction

Seismicity is generally thought to comprise a mixture of a background process that expresses the continuum of tectonic deformation in a given seismogenetic area and a population of aftershock sequences (foreground process) that express the short-term activity associated with significant background events. The statistical physics of background seismicity, hence the nature of the seismogenetic system, is not clear. In consequence, the way in which seismicity and tectonic deformation evolve is also not well understood, with significant repercussions on problems such as hazard analysis and long-term forecasting.

There are two principal approaches toward understanding the statistical physics of seismicity. The first and currently most influential, postulates that the expression of the background process is Poissonian in time and space and obeys *extensive* Boltzmann–Gibbs thermodynamics. It is important to emphasize that this property is associated only with the time and the distance (space) between earthquake events, but not with their size (magnitude) which is governed by the well-established frequency–magnitude (F–M)

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relationship of Gutenberg and Richter.[1] Paradigmatic expression of this viewpoint is the Epidemic Type Aftershock Sequence (ETAS) (e.g. Ogata, 1988, 1998; Zhuang et al., 2002; Helmstetter and Sornette, 2003; Touati et al., 2009; Segou et al., 2013). In this empirical construct which essentially expresses a self-excited conditional Poisson process (Hawkes, 1972; Hawkes and Adamopoulos, 1973; Hawkes and Oakes, 1974), the randomly occurring background main events trigger their aftershock sequences in which aftershocks trigger their own sub-sequences thus leading to short-term clustering of multiple generations of foreground events; these are dependent on each other and their time dependence is described by a power law known as the Omori–Utsu law of aftershocks (e.g. Utsu et al., 1995). There are also point process models developed to address the problem of intermediate to long-term clustering, as for instance the EEPAS (Each Earthquake is a Precursor According to Scale, e.g. Rhoades, 2007) and the PPE (Proximity to Past Earthquakes, e.g. Marzocchi and Lombardi, 2008). In any case, point processes are memoryless, therefore at the core of this viewpoint rests the assumption that background earthquakes are statistically independent and although it is possible for one event to trigger another (smaller or larger), this occurs in an unstructured random fashion and does not to contribute to the long-term evolution of seismicity.

The second viewpoint postulates that the seismogenetic process comprises a complex system, although the mechanisms begetting complexity are not clear as yet. A well-studied class of models (Bak and Tang, 1989; Sornette and Sornette, 1989; Olami et al., 1992; Sornette and Sammis, 1995; Rundle et al., 2000; Bak et al., 2002; Bakar and Tirnakli, 2009; many others) suggests that seismicity expresses a non-equilibrating fractal tectonic grain that continuously evolves toward a stationary critical condition with no characteristic spatiotemporal scale (Self Organized Criticality – SOC). In this view, all earthquakes belong to, or evolve toward the same global population and participate in shaping a non-equilibrium state in which events develop spontaneously and any small instability has a chance of cascading into a large shock.

Critical complex systems evolving in a fractal-like space–time are characterized by long-range interactions and long-term memory which, at least at a regional scale, should be manifested by correlations and power-law distributions observable in the statistical behaviour of their energy release, temporal dependence and spatial dependence. In addition, there are models proposing alternative complexity mechanisms that do not involve criticality, yet maintain the seismogenetic system in a state of non-equilibrium (see Sornette and Werner, 2009 for a discussion). More recently, the Coherent Noise Model (Newman, 1996) was applied to seismicity by Celikoglu et al. (2010). This is based on the notion of external stress acting coherently onto all agents of the system without having any direct interaction with them and is shown to generate power-law interevent time distributions. A weak point in this model is that it does not include some geometric configuration of the agents and it is not known how this would influence the behaviour of the system.

A fundamental difference between the Poissonian models and SOC is their understanding of the background seismogenetic process. The former approach assumes a self-exciting Poisson process in time and space, in which there is no correlation (interaction) between background events, so that the statistical description of parameters pertaining to their temporal and spatial evolution would be consistent with the Boltzmann–Gibbs formalism. The SOC formalism requires short and long-range interactions in a non-equilibrium state, so that there would be *correlation* between background events, as well as between background/foreground events and foreground/foreground events. This renders memory to the system and the statistics of the parameters pertaining to its temporal and spatial evolution are expected to exhibit power-law behaviour and long tails. Moreover, non-critical complexity models cannot develop power-law distributions in space and time, unless they evolve in non-equilibrium states. Poissonian models and SOC both agree that the foreground process (aftershock sequences) comprise a set of dependent events, but whereas the former assign only local significance to this dependence, SOC considers them to be an integral part of the regional seismogenetic process.

The above discussion makes clear that were it possible to identify and remove the foreground process (aftershocks), it might also be possible to clarify the nature and dynamics of the background process by examining its spatiotemporal characteristics for the existence of correlation (hence non-extensivity). This is not a simple objective and before it is pursued, there must be satisfactory answers to three basic requirements, which are: (a) Statistical physics that comprise a *natural* and befitting (*not* model-based) general context in which to investigate the existence of correlation; (b) Appropriate parameters for the analysis of correlation in the temporal and spatial properties of seismicity and, (c) Effective ways of distinguishing the background from the foreground processes. It turns out that satisfactory (or nearly satisfactory) answers exist, as will be elaborated forthwith.

The most recent development in the statistical description of earthquake occurrence is the introduction of Non Extensive Statistical Physics (NESP) as a fundamental conceptual framework of the thermodynamics that govern seismogenesis and seismicity. NESP has been developed by Tsallis (1988, 2009) as a generalization of the (extensive) Boltzmann–Gibbs formalism to non-extensive (non-equilibrating) systems. As such it comprises an appropriate tool for the analysis of complexity evolving in a fractal-like space–time and exhibiting scale invariance, long-range interactions and long-term memory (e.g. Gell'mann and Tsallis, 2004). NESP predicts power-law cumulative probability distributions for non-extensive (complex) dynamic systems, which reduce to the exponential cumulative distribution in the limiting case of extensive (random) systems. Thus, NESP provides a unique, consistent and *model-independent* theoretical context in which to investigate the nature and dynamics of the background and foreground seismogenetic processes.

With respect to the second requirement above, a common measure of scaling in earthquake size is the Gutenberg–Richter F–M distribution, which is interpreted to express the scale-free statistics of a fractal active tectonic grain. The F–M distribution is *static* and does not say much about the temporal dynamics (evolution) and spatial dynamics of the seismogenetic system. It also says nothing about correlation in the characteristics of energy release, since it does not relate the energy released by a given earthquake to the energy released by its predecessor or successor events. Nevertheless, this undisputable empirical relationship is a yardstick against which to compare any physical and statistical description of the relationship between earthquake size and frequency and as such will be used herein.

A measure of the temporal dynamics and definite measure of possible correlation between successive earthquakes is the time elapsed between consecutive events above a magnitude threshold over a given area: this parameter is variably referred to as *interevent time*, *waiting time, calm time* etc. Understanding the statistics of the earthquake frequency – interevent time (F–T) distribution is obviously of paramount importance for understanding the dynamics of seismogenetic system and have been studied by several researchers, albeit not as extensively as its F–M counterpart. The empirical F–T distributions generally exhibit power-law

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[1] An apparent contradiction is that the scale-free grading between earthquake frequency and magnitude implied by the F–M relationship cannot be derived from the Boltzmann–Gibbs formalism.