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The application of computed piezomagnetic models to volcanic phenomena

Shin'ya Sakanaka *

Department of Earth Science and Technology, Faculty of Engineering and Resource Science, Akita University, 1-1, Tegata-Gakuen-Machi, Akita 010-8502, Japan

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ABSTRACT

Piezomagnetic models, suitable for volcanological applications, can be readily calculated by numerical surface integral methods, providing both savings in computational time and an ability to deal with arbitrarily shaped magnetoelastic media. Studies of piezomagnetic changes have traditionally used models based on the Mogi model in which a spherical underground pressure source is included to represent a magma chamber. We extended the Mogi model to include inclined column sources, as in the Walsh and Decker model, and evaluated piezomagnetic changes for vertical, horizontal, and inclined columns. Our analysis method considers any dependency of magnetic changes on the angle of column inclination. This numerical approach allows for construction of piezomagnetic models that closely resemble actual volcanic phenomena.

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1. Introduction

Crustal activity produces geomagnetic changes. The mechanisms involved can result from stress or from geothermal and electrokinetic activity. Stress-induced magnetism, also known as piezomagnetism, is an important mechanism responsible for geomagnetic changes and is commonly associated with volcanic activity and earthquakes.

In early piezomagnetic models, converting changes in stress to changes in magnetization was quite complicated, particularly in evaluating the piezomagnetic effect. The results of laboratory experiments (e.g., Kalashinikov and Kapitsa, 1952; Ohnaka and Kinoshita, 1968) enabled Sasai (1983) to develop a relationship linking stress to initial magnetization. This relationship, analogous to Hooke's law of elastic behavior, has been widely used in piezomagnetic modeling. Sasai (1983) referred to it as the "linear piezomagnetic effect."

Piezomagnetic modeling can be applied to both earthquake and volcanic activity. Piezomagnetism resulting from fault dislocation is known as the seismomagnetic effect (Stacey, 1964), while that resulting from volcanic activity is referred to as the volcanomagnetic effect (Stacey et al., 1965). Both terms are contained within the general concept of piezomagnetism.

Stacey (1964) introduced seismomagnetic modeling that employed a numerical volume element procedure, and several subsequent studies employed that method (e.g., Shamsi and Stacey, 1969; Talwani and Kovach, 1972; Hildenbrand, 1975; Zlotnicki and Cornet, 1986). An alternative analytical approach was applied in a series of papers by Sasai (e.g., Sasai, 1980, 1991b, 1994). Utsugi et al. (2000) then presented an analytical expression for the piezomagnetic field generated by dislocation of inclined faults.

In volcanomagnetic modeling, Davis (1976) followed Stacey et al. (1965) and derived a numerically computed model using a simulated magma chamber. An analytical approach was preferred by Sasai (1979) who applied the point source solution offered by the so-called Mogi model (Mogi, 1958). This utilizes an inflated or deflated pressure source within a semi-infinite magnetoelastic medium. A discrepancy existed between Sasai's (1979) and Davis' (1976) results, and subsequently Suzuki and Oshiman (1990) rederived Davis' (1976) Mogi model solution by means of a volume element approach that varied the depth of the Curie point isotherm. Their result prompted Sasai (1991a) to revise his analytical solution of the Mogi model.

Numerical approaches to piezomagnetic modeling have traditionally employed a volume element method or a volume integral of the dipole forces existing within a magnetoelastic media. It is also possible to use the surface integral approach introduced by Sasai (1983). Such a surface integral formula as applied to a magnetoelastic media provides a "representation theorem" of piezomagnetism. Sakanaka et al. (1997) applied such a representation theorem in numerical piezomagnetic calculations of a two-dimensional magnetoelastic medium. Such numerical calculations save computational times in comparison with the numerical volume integral methodology.

Advantages of numerical approaches lie in their ready application to practical problems. Numerical approaches can be applied

* Tel.: +81 18 889 2373; fax: +81 18 837 0401.
E-mail address: sakanaka@ipc.akita-u.ac.jp

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to an arbitrarily shaped magnetoelastic medium in which magnetization is arbitrarily distributed. Sakanaka et al. (1997) derived the volcanomagnetic solution for a two-dimensional Mogi model (the "Yukutake model" (Yukutake and Tachinaka, 1967)) that contained jagged topography. In a three-dimensional case, Yamazaki and Sakai (2006) evaluated the piezomagnetic field arising in a Mogi model having a cone-shaped volcanic topography. Earlier, Oshiman (1990) had used a two-dimensional model to describe the non-uniform distribution of initial magnetization arising from volcanomagnetic effects.

2. Theory

This study describes examples of models of piezomagnetism produced by volcanic phenomena using three-dimensional numerical surface integral techniques introduced by Sakanaka (1998). To deduce the piezomagnetic field, we must first establish the distribution of the strain or stress changes within each geophysical situation (cf. the Mogi model). To do so, we could use the numerical solutions calculated for the resulting strain field by either the finite element method (FEM) or boundary element method (BEM). In this study, however, analytical solutions of strain are used to describe the piezomagnetic field.

Once the stress field has been determined, the numerical surface integral approach based on the representation theorem is applied. The representation theorem denoting the piezomagnetic scalar potential $W_k(\boldsymbol{r})$ originated from the kth component of initial magnetization J_k at a position $\boldsymbol{r}$, outside of a magnetoelastic body in the Cartesian coordinate (x_1,x_2,x_3), is given as

$$W_k(\boldsymbol{r}) = \int\int_s \left[\beta\mu(\mathbf{1}+\nu)\left\{-\frac{\partial \mu_{\boldsymbol{k}}(\boldsymbol{r}')}{\partial \boldsymbol{n}'}\left(\frac{\mathbf{1}}{\rho}\right) + \boldsymbol{u_k}(\boldsymbol{r}')\frac{\partial}{\partial \boldsymbol{n}'}\left(\frac{\mathbf{1}}{\rho}\right)\right\}\boldsymbol{J_k} + \frac{\boldsymbol{\Delta M_k}\cdot\boldsymbol{n}'}{\rho}\right]\boldsymbol{dS}. \quad (1)$$

$$\rho = |\boldsymbol{r}-\boldsymbol{r}'| = \sqrt{(x_1-x_1')^2+(x_2-x_2')^2+(x_3-x_3')^2}.$$

$$\frac{\partial}{\partial n'} = n_1'\frac{\partial}{\partial x_1'} + n_2'\frac{\partial}{\partial x_2'} + n_3'\frac{\partial}{\partial x_3'}.$$

$$\Delta M_{kl} = \beta\mu\left\{-\delta_{kl}\mathrm{div}\boldsymbol{u}(\boldsymbol{r}') + \frac{\mathbf{3}}{\mathbf{2}}\left(\frac{\partial \boldsymbol{u_k}}{\partial \boldsymbol{x_l'}} + \frac{\partial \boldsymbol{u_l}}{\partial \boldsymbol{x_k'}}\right)\right\}\boldsymbol{J_k}.$$

Eq. (1), following Sasai (1983), employs Gaussian CGS units. The traditional Gaussian expressions are somewhat simpler to use, especially when magnetization terms are included, than are International System (SI) units. The potential $W_k(\boldsymbol{r})$ with suffix k originates from the kth component of the initial rock magnetization J_k. Primes on terms (e.g., $\boldsymbol{r}', \boldsymbol{r}'$) represent the quantities on the boundary surface of the body. ρ is a distance between a point on the surface and the observation position. u_k is the kth component of the displacements, and $\boldsymbol{r}'(=(n_1', n_2', n_3'))$ is an outward normal vector on the surface. The partial derivative of displacement corresponds to the strain component. $\boldsymbol{\Delta M}_k$ is a vector of magnetization change originating from the kth component of the initial magnetization, and ΔM_{kl} is the lth component of ΔM_k. β is the stress sensitivity related to changes in magnetization arising from the stress component. δ_{kl} is the Kronecker delta.

To calculate the numerical surface element integral, the surface of a magnetized body is discretized into triangular or rectangular (trapezoidal) elements. If $\partial u_k(\boldsymbol{r}')/\partial n', u_k(\boldsymbol{r}'), \boldsymbol{\Delta M_k}$, and $\boldsymbol{r}'$ in Eq. (1) are assumed to be constant for one element, then the piezomagnetic field, $\Delta H_i(\boldsymbol{r})$, in the direction of the ith axis is

$$\begin{aligned}\Delta H_i(\boldsymbol{r}) &= -\frac{\partial}{\partial x_i}\sum_{k=1}^{3} W_k(\boldsymbol{r}) \\ &= \int\int_s \sum_{k=1}^{3}\left[\frac{\beta\mu(1+\nu)}{\rho^3}\left\{-\frac{\partial u_k(\boldsymbol{r}')}{\partial n'}(x_i-x_i') + u_k(\boldsymbol{r}')n_i' - \frac{3u_k(\boldsymbol{r}')(x_i-x_i')}{\rho^2}\sum_{j=1}^{3}(x_j-x_j')n_j'\right\}J_k + \frac{\boldsymbol{\Delta M}_k\cdot n'}{\rho^3}(x_i-x_i')\right]\mathrm{d}S.\end{aligned} \quad (2)$$

The right-hand side of Eq. (2) contains the sum of the piezomagnetic scalar potentials originating from three components of the initial magnetization. The north component of the piezomagnetic field ΔX, the east component ΔY, and the vertically downward component ΔZ can be directly calculated using Eq. (2). The changes in the geomagnetic total intensity ΔF are obtained by

$$\Delta F = \Delta X\cos D\cos I + \Delta Z\sin I. \quad (3)$$

where I is the inclination and D is the declination of the geomagnetic field at an observation point.

3. Traditional volcanomagnetic models

The Mogi model commonly uses a simulated magma chamber to analyze volcanological-induced effects, as shown in Fig. 1, with changes in hydrostatic pressure assumed to occur within the spherical body buried underground. Mogi (1958) evaluated the effects of crustal deformation, and other studies have calculated changes in gravity (Hagiwara, 1977) and the magnetic field (Sasai, 1979, 1991a). The analytical solution of the relevant displacements is given in Appendix A.

The two-dimensional version of the Mogi model, called the "Yukutake model," was introduced by Yukutake and Tachinaka (1967) to allow for numerical estimations of magnetic changes. The Yukutake model assumes an infinitely long cylindrical pressure source within a semi-infinite magnetoelastic medium. Solutions of the Yukutake model have been given in detail by Oshiman (1990) and Sakanaka et al. (1997).

Fig. 2 presents schematic diagrams of the discretized surface using the numerical computations of the present study for both the Mogi and Yukutake models. The surface of the magnetoelastic medium consists principally of four parts: (1) the ground surface, (2) the surface around the pressure source, (3) the side walls, and (4) the Curie point isotherm. For simplicity, only the surface around the pressure source is shown in Fig. 2; this coincides with the outer surface of the demagnetized area due to high temperature. In the volcanomagnetic models used in this study, the ground surface is flat, i.e., it lacks topography. The Curie point isotherm is

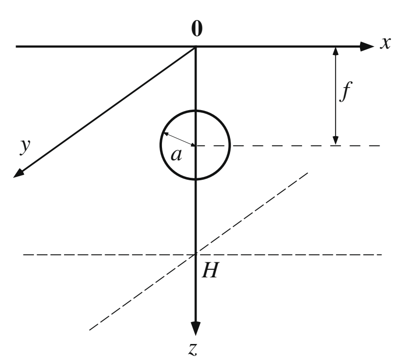

Fig. 1. The coordinate system of the Mogi model. The center of a hydrostatically pumped spherical source with radius a is located at the point (0,0,f). The magnetized area occupies the region between the ground surface and the depth of the Curie point isotherm, H, except for the interior of the source.

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