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Mechanical modeling of MX-80 – Development of constitutive laws

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ABSTRACT

In a previous paper [Åkesson, M., Hökmark, H., 2007. Mechanical model for unsaturated MX-80. In: Schanz, T. (Ed.), Theoretical and Numerical Unsaturated Soil Mechanics, Springer Proceeding in Physics, vol. 113, pp. 3–10] a new elastoplastic model for unsaturated swelling clay was presented. The model was formulated as two differential equations, for elastic and plastic strains, respectively. Each equation described a relation between the (air-filled) macro void ratio and the two independent variables stress and (water-filled) micro void ratio. The established concept of swelling pressure and its void ratio dependence were incorporated in the model. To do this, the grain-to-grain contact stress and the contact area had to be considered. In its original form the model only handled one-dimensional problems, e.g. compression tests with uniaxial strain and water uptake test at constant volume with assumed isotropic conditions.

This paper describes a first attempt to generalize the model to address multidimensional problems with the intention to integrate it into the framework of Code_Bright. This was made through definition of an expression for the retention properties and through variable substitution. In this way, four functions for the kappa moduli were identified: two elastic and two plastic. These functions provide unique values for each given state. In the used framework of the Barcelona Basic Model (BBM), all strains are treated as elastic, and there is thus no explicit treatment of any deviatoric yield condition. The only remaining parameter outside the new model is the Poisson's ratio.

1. Introduction

Thermo-Hydro-Mechanical modeling of buffer and backfill material is an important subject in the Swedish nuclear waste disposal program and Code_Bright is one of the codes employed in that program. Thermoelastoplastic constitutive laws, based on the BBM (Alonso et al., 1990), which in turn is an extension of the modified Cam–Clay model (Roscoe and Burland, 1968), are used in the code for the description of the mechanical behaviour of compacted bentonite. BBM has, however, certain limitations for expansive soils, and has therefore been further developed into the Barcelona Expansive Model (BExM) (Alonso et al., 1999).

An alternative approach to describe the mechanical behaviour of swelling clays, in particular MX-80 bentonite, has recently been presented (Åkesson and Hökmark, 2007). This paper describes a first attempt to generalize this model, from hereon denoted the *Bentonite Elastoplastic Swelling pressure and Contact stress based* (BESC) model, to address multidimensional problems with the intention to integrate it into the framework of Code_Bright. The basic assumptions, the mathematical formulation and the parameter settings of the BESC model are first described briefly. The method of embedding this model in the BBM framework is thereafter presented. Application examples for compression and swelling tests with uniaxial strain (oedometers), as well as water uptake tests at constant volume are shown in the subsequent section. Finally, the current limitations of the BESC model are discussed.

2. Short description of the BESC model

The model is based on the following assumptions:

i. The porosity is two-parted in micro- and macro-porosity. The void ratio e is thus the sum of a micro (e_m) and a macro (e_M) void ratio. The same approach is used in BExM (Alonso et al., 1999).
ii. The micro-porosity is water-filled, whereas the macro-porosity is air-filled. This is basically a definition, although it can be interpreted as if all water is adsorbed in interlayer pore space. This is supported by the high capacity of water uptake observed for sodium bentonite (e.g. Norrish, 1954). The micro void ratio is thus defined as $e_m = V_w/V_s = w \cdot \rho_s/\rho_w$, where w is the water content and ρ_s/ρ_w is the particle-water density ratio (=2.78).
iii. The two void ratios (e_m and e_M) and the stress (σ), are the state variables of the model (σ is positive for compressive stresses). The stress is defined as a scalar, denoting the axial

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stress in 1D oedometer tests, and the hydrostatic pressure for isotropic problems. The model is extended through identification of σ with the net mean stress and introduction of a Poisson's ratio (see Section 3).

iv. The contact stress (σ_c) between grains is defined by a ratio, $\alpha = \sigma/\sigma_c$, corresponding to the ratio between the contact area and the section area. α is assumed to be a function of e_m and e_M.
v. The swelling pressure (p_s) is a central quantity of the model, and is determined by the micro void ratio only.
vi. A condition for elastic strains is that $\sigma_c < p_s$, while $\sigma_c = p_s$ for plastic strains. A transition from elastic to plastic strains is made when the latter condition is fulfilled. No deviatoric condition for plastic strains has yet been defined.
vii. The elastic domain is governed by a modulus of compression (K), determined by the micro void ratio only.

The elastic relation can be expressed as:

$$\left[\frac{\sigma}{\alpha}\cdot\frac{\partial\alpha}{\partial e_M} - \frac{\alpha\cdot K}{1+e_m+e_M}\right]\cdot de_M = d\sigma - \frac{\sigma}{\alpha}\cdot\frac{\partial\alpha}{\partial e_m}\cdot de_m \tag{1}$$

The plastic relation can be expressed as:

$$\frac{\sigma}{\alpha}\cdot\frac{\partial\alpha}{\partial e_M}\cdot de_M = d\sigma - \left[\alpha\cdot\frac{dp_s}{de_m} + \frac{\sigma}{\alpha}\cdot\frac{\partial\alpha}{\partial e_m}\right]\cdot de_m \tag{2}$$

The functions used in the model for the modulus of compression, swelling pressure and the contact area, respectively, are given as follows:

$$K(e_m) = K_1\cdot\exp(5\cdot(1-e_m))\ \text{(MPa)} \tag{3}$$

$$p_s(e_m) = 2\cdot e_m^{\frac{-1}{0.24}}\ \text{(MPa)} \tag{4}$$

$$\alpha(e_m, e_M) = \left(\frac{1+e_m}{1+e_m+e_M}\right)^{\gamma}(-) \tag{5}$$

The values of the parameters K_1 and γ were in this study the same as for isotropic 3D conditions used in the original paper: K_1 = 50 MPa and γ = 15.

3. Embedment of the BESC model in the BBM framework

The elastic part of the BBM model is defined as:

$$\begin{aligned} -de &= \kappa_i\cdot\frac{dp'}{p'} + \kappa_s\cdot\frac{ds}{s+0.1} \\ d\boldsymbol{\varepsilon}_d &= \frac{1}{2G}\cdot d\boldsymbol{\sigma}_d, \quad G = \frac{3(1-2v)}{2(1+v)}\frac{(1+e)p'}{\kappa_i} \end{aligned} \tag{6}$$

where $\kappa_i = \kappa_i(p', s, e)$ and $\kappa_s = \kappa_s(p', s, e)$ are the elastic modulus functions through which the BESC model is embedded in the BBM framework. In the equation above e is the void ratio, p' is the net mean stress, s the suction, $\boldsymbol{\varepsilon}_d$ the deviatoric strain, $\boldsymbol{\sigma}_d$ the deviatoric stress, v Poisson's ratio and G the shear-modulus.

The translation of the BESC model into a state-specific kappa modulus requires a consistent description of retention properties since this model uses e_m as a state variable rather than suction. The actual suction, s, is calculated using: $s = \Psi(e_m) - p'$ (Dueck, 2007), where ψ is suction for free swelling conditions, here identified with the defined swelling pressure function, Eq. (4). With this relation, the state variables e, p' and s can be substituted for e_m, e_M and σ:

$$\begin{cases} e_m = p_s^{-1}(s+p') \\ e_M = e - p_s^{-1}(s+p') \\ \sigma = p' \end{cases} \tag{7}$$

The state, elastic or plastic, is determined by the conditions $\sigma_c < p_s$ and $\sigma_c = p_s$, respectively. The kappa modulus functions are defined for elastic, $(\kappa_i, \kappa_s)^e$, and plastic conditions, $(\kappa_i, \kappa_s)^p$, and provide unique values for each given state. In the used framework of BBM, all strains are treated as elastic, and there is thus no explicit treatment of any deviatoric yield condition. The only remaining parameter outside the BESC model is the Poisson's ratio, v.

In order to derive the kappa modulus functions the suction is differentiated as:

$$ds = dp_s - dp' = \frac{dp_s}{de_m}\cdot de_m - dp' \tag{8}$$

With the condition that either s or p is constant; the kappa moduli can be identified from the Eqs. (1), (2), and (8) and the following expressions:

$$\begin{cases} de_m + de_M = -\kappa_i\cdot\dfrac{dp'}{p'} & (ds = 0) \\ de_m + de_M = -\kappa_s\cdot\dfrac{ds}{s+0.1} & (dp' = 0) \end{cases} \tag{9}$$

The resulting four functions are shown in Table 1.

In order to implement the embedded model in the source code of Code_Bright, it is essential that this is made concurrently with the employment of a retention curve consistent with the mechanical model. This can be achieved through expressing the retention properties given by Eq. (7) in terms of degree of saturation (S_l):

$$S_l(p', s, e) = \frac{p_s^{-1}(s+p')}{e} \tag{10}$$

4. Test of embedded model

The embedded model has been tested for compression tests with uniaxial strain and constant relative humidity (Fig. 1), swelling tests with uniaxial strain and constant axial load (Fig. 2) and isotropic swelling pressure build-up (Fig. 3). The compression tests and the swelling tests were made in oedometers, consisting of a steel ring around the sample with filters on both sides (Dueck, 2007). The samples were exposed to humidified air, with specified relative humidity, through grooves above and below the filters. Pistons and force transducers were placed axially above the sample, as well as radially through the steel ring. The axial as well as the radial stresses could thereby be measured, although no radial strains were allowed. Calculations were made with simple routines written in the MathCad® environment, following the equation system presented in an accompanying paper (Kristensson and Åkesson, 2008). The models of the compression and the swelling tests were chosen to simulate experimental results presented by Dueck and Nilsson (2008). In these calculations, v were fitted to match the experimental results. The model of the swelling pressure build-up at constant volume was generic.

The compression tests shown in Fig. 1, were performed at 24 and 28 MPa suction, respectively. With the employed description of the retention properties, Eq. (7), this corresponds to water content of approx. 19%, which was also found in the analysis after the tests. The slopes during compression as well as the apparent pre-

Table 1
Kappa modulus functions.

	Elastic (Condition: $p' < \alpha\cdot p_s$)	Plastic (Condition: $p' = \alpha\cdot p_s$)
κ_i	$-p'\cdot\left[\frac{1}{\frac{dp_s}{de_m}} + \frac{1-\frac{p'}{\alpha}\cdot\frac{\partial\alpha}{\partial e_m}\cdot\frac{1}{\frac{dp_s}{de_m}}}{\frac{p'}{\alpha}\cdot\frac{\partial\alpha}{\partial e_M} - \frac{\alpha\cdot K}{1+e_m+e_M}}\right]$	$-p'\cdot\left[\frac{1}{\frac{dp_s}{de_m}} + \frac{1-\alpha-\frac{p'}{\alpha}\cdot\frac{\partial\alpha}{\partial e_m}\cdot\frac{1}{\frac{dp_s}{de_m}}}{\frac{p'}{\alpha}\cdot\frac{\partial\alpha}{\partial e_M}}\right]$
κ_s	$-(s+0.1)\cdot\left[\frac{1}{\frac{dp_s}{de_m}} - \frac{1-\frac{p'}{\alpha}\cdot\frac{\partial\alpha}{\partial e_m}\cdot\frac{1}{\frac{dp_s}{de_m}}}{\frac{p'}{\alpha}\cdot\frac{\partial\alpha}{\partial e_M} - \frac{\alpha\cdot K}{1+e_m+e_M}}\right]$	$-(s+0.1)\cdot\left[\frac{1}{\frac{dp_s}{de_m}} - \frac{\alpha\cdot\frac{dp_s}{de_m} + \frac{p'}{\alpha}\cdot\frac{\partial\alpha}{\partial e_m}}{\frac{p'}{\alpha}\cdot\frac{\partial\alpha}{\partial e_M}\cdot\frac{dp_s}{de_m}}\right]$