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Restoration of blurred color images with impulse noise[☆]

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ABSTRACT

Restoration of images degraded by blurring and impulse noise has received considerable attention recently. Guo et al. (2009) proposed a fast l_1-total variation algorithm for grayscale image restoration with impulse noise. In this paper, we extend their idea for deblurring color images with impulse noise. An alternating iteration scheme is adopted for solving the corresponding problem. More importantly, we employ the five-point property to analyze the convergence of the proposed alternating algorithm. Numerical experiments demonstrate that the proposed method could deblur color images with good quality.

1. Introduction

In the area of remote sensing, materials science, medical and astronomical imaging and so on, image restoration plays an important role in preprocessing and post-processing the image. Recently, the color image restoration problem has attracted much attention due to its wide range of applications [1–4]. The perception of color is of great importance to human beings since we use color features to sense the environment, recognize objects and convey information.

In many situations, the recording process may cause some undesirable consequences such as blurring and noise on the observed images. For instance, they may be blurred because of motion and out-of-focus etc., and may be also corrupted by noise due to errors generated in noisy sensors or communication channels. In this work, we are interested in the problem of restoring color image degraded by blur and impulse noise.

Unless stated otherwise, we assume that the underlying images are in the RGB color model and have square domains and let the original color image $\boldsymbol{f} = (f_r; f_g; f_b) \in \mathbb{R}^{3n^2}$, where each of the n^2-length vectors f_i $(i \in \{r, g, b\})$ results from the lexicographic ordering of the two-dimensional signals in each channel. Mathematically, image degradation caused by blur and impulse noise can be modeled as

$$\boldsymbol{g} = N_{imp}(\boldsymbol{Hf}), \tag{1}$$

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where $\boldsymbol{g}$ represents the observed image, N_{imp} models the corruption of impulse noise and the blurring matrix $\boldsymbol{H}$ is of the form

$$\boldsymbol{H} = \begin{pmatrix} H_{rr} & H_{gr} & H_{br} \\ H_{rg} & H_{gg} & H_{bg} \\ H_{rb} & H_{gb} & H_{bb} \end{pmatrix} \in \mathbb{R}^{3n^2 \times 3n^2}, \tag{2}$$

where the diagonal blocks represent the within-channel degradation, and the off diagonal blocks stand for the cross-channel degradation.

An important feature of the image contaminated by impulse noise is that only a certain part of pixels of the image are corrupted and the remaining ones are noise free. A variety of techniques have been proposed for the removal of impulse noise, such as the vector median filter [5] and vector directional filter [6,7]. There are also methods for noise detection with the noise removal [8–10] which is out of scope here. Unfortunately, most of these filters were designed for denoising only and not suitable for deblurring [11]. As is well known, the image restoration problem is frequently ill conditioned, and thus directly solving (1) will yield a solution that is extremely sensitive to noise. Therefore, regularization methods are good choices for getting a stable and accurate solution. A popular regularization approach to overcome ill-conditioning dates back to the works of Tikhonov [12]. Although the minimization problem related to Tikhonov regularization is easy to handle, it tends to produce over smoothed images and fails to adequately preserve important image attributes such as sharp edges [13]. In order not to overly penalize the discontinuities during the restoring process, Rudin, Osher and Fatemi proposed the total variation (TV) regularization [14] which has a wide range of applications [15–17].

Before we deal with color images, we briefly introduce a common function involving l_1 data-fitting and TV regularization for gray-scale image restoration. The formulation is as follows

$$\min_f \{\|Hf - g\|_1 + \lambda \|f\|_{TV}\} \tag{3}$$

where $H \in \mathbb{R}^{n^2 \times n^2}, f \in \mathbb{R}^{n^2}$ and $g \in \mathbb{R}^{n^2}$ are the blurring matrix, the true image and the degraded image, respectively. λ is the positive regularization parameter that plays the role of balancing the importance between the data-fitting term $\|Hf - g\|_1$ and the regularization term $\|f\|_{TV}$. The term $\|\cdot\|_{TV}$ denotes the discrete TV seminorm of f which is given by

$$\|f\|_{TV} := \sum_{1 \le i \le n^2} |(\nabla f)_i| = \sum_{1 \le i \le n^2} \sqrt{|(\nabla f)_i^x|^2 + |(\nabla f)_i^y|^2},$$

where $|\cdot|$ is the Euclidean norm in $\mathbb{R}^2$. This kind of TV-norm is usually referred to as the isotropic TV. Following Zhu et al.'s work in [18], we define the discrete gradient operator $\nabla : \mathbb{R}^{n^2} \to \mathbb{R}^{n^2 \times 2}$ as follows:

$$(\nabla f)_i = ((\nabla f)_i^x, (\nabla f)_i^y), \quad i = 1, 2, \ldots, n^2$$

with

$$(\nabla f)_i^x = \begin{cases} f_{i+1} - f_i, & \text{if } i \bmod n \neq 0, \\ 0, & \text{if } i \bmod n = 0 \end{cases}$$

and

$$(\nabla f)_i^y = \begin{cases} f_{i+n} - f_i, & \text{if } i \le n^2 - n, \\ 0, & \text{if } i > n^2 - n. \end{cases}$$

Many techniques for solving the l_1 data-fitting problem (3) were proposed. A splitting method was proposed in [19], which introduced two auxiliary variables in order to replace the l_1 data-fitting problem with an l_2-deblurring (sub-)problem that can be solved easily. A primal dual active set algorithm has been investigated in [20]. The duality-based splitting method was proposed in [21]. We refer the reader to [22,20,23] and the references therein for more details involving the l_1 data-fitting problems. Some other impulse noise removal methods can be found in [24,25].

To exploit the edge-preserving feature of the color image, Blomgren et al. [26] and Bresson et al. [27] extended the gray-level TV norm to the "color TV" (CTV) norm and the vectorial TV norm (VTV), respectively. The VTV is also called as MTV which is short for multichannel TV and widely used in the literature [28,11,27]. It has been shown that both CTV and VTV preserve the two desirable properties of not overly penalizing discontinuities in the image and being rotationally invariant [28]. Since CTV and VTV give restoration of similar quality [23] and there is a fast algorithm for VTV regularization proposed by Bresson and Chan in [27], we choose the VTV in this paper. The discrete form of VTV is shown below:

$$\|\boldsymbol{f}\|_{VTV} := \sum_{1 \le l \le n^2} \sqrt{\sum_{k \in \{r,g,b\}} |(\nabla \boldsymbol{f}_k)_l^x|^2 + |(\nabla \boldsymbol{f}_k)_l^y|^2}.$$

Like the regularization function in (3) for the gray-scale image restoration, we establish a regularization function for the color image restoration

$$\min_{\boldsymbol{f}} \{\|\boldsymbol{H}\boldsymbol{f} - \boldsymbol{g}\|_1 + \lambda \|\boldsymbol{f}\|_{VTV}\}, \tag{4}$$

where $\boldsymbol{H}$ is the blurring matrix which has been defined in (2) and λ is the regularization parameter.