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Weighted curvature-preserving PDE image filtering method

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ABSTRACT

The tensor-driven curvature-preserving partial differential equation is an outstanding anisotropic diffusion filtering model. To effectively preserve image edge well, a weighted curvature-preserving PDE based filtering method is proposed, which employ local image directional information to design weight coefficient for different vector fields. Experimental results indicate that new approach shows superior performance on preserving image edge and curvature geometric structure.

1. Introduction

Filtering is an important research subject in image processing. On one hand, it can suppress noise effectively, providing exact information for further processing including image segmentation, edge detection, and target identification. On the other hand, research involving filtering methods can promote image regularization and image model theories [1,2]. The existing filtering methods are mainly based on two ideas: (1) sparse decomposition [3–5] and (2) smoothing filtering [6–8]. The former regards the useful information as sparse constituents in images and noise as residuals of the images. In these methods, the construction of a sparse dictionary is one of the key problems. The latter views noise as a local oscillation signal which can be removed by the smoothing filtering. Constructing the image structure description operator is the crucial point of such methods. The sparse decomposition filtering method requires a comparatively longer computation time. This paper focuses on the smoothing filtering method.

Recently, research on the smoothing filtering method has mainly concentrated on two aspects: (1) diffusion PDE filtering methods [9–16] and (2) Non-local Mean Filtering (NLMF) [17,18]. The former is a local method and the latter is a non-local method. Local-mean filtering methods have developed rapidly in the last 20 years and they have been successfully applied to many fields of computer vision. In recent years, non-local filtering methods have gradually become a hot research topic due to its features such as simple equations, no iterations, and good performance. Although, this kind of method may easily cause a staircase effect in pictures with a slow changing gray scale such as images of people's faces. Therefore, for searching similar picture pieces and computing their weight coefficients, the non-local methods consume more time preventing their practical applications.

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The research of diffusion PDE filtering methods has evolved from linearity to nonlinearity as well as isotropy to anisotropy. These methods mostly use the theory of multi-scale analysis, designing adaptive diffusion PDE methods from directional filtering view point. Adaptive diffusion PDE filtering methods can be divided into two methods: isotropic and anisotropic. A representative of the former is the PM filtering method [9] and an example of the latter is tensor-driven PDEs [10–12]. PM filtering is non-linear and isotropic and it can obtain better results when processing segmented smooth pictures such as cartoon images, indoor images, and medical images.

However, the above-mentioned methods are unfit for images with rich structures as the PM method does not consider the local structure directional information, which is isotropic when smoothing pictures. Therefore, the anisotropic tensor-driven PDE filtering method has attracted much attention.

In a study of tensor-driven PDE image filtering methods, Weickert et al. [10] introduced diffusion tensors in a diffusion PDE and proposed the Divergence-based Tensor-driven PDE (DT-PDE). This method utilizes structure tensors as the local structure description operators and constructs diffusion tensors by structure tensors, which makes the diffusion speed faster at the direction of the image edges but slower at the directions perpendicular to the image edges. Tschumperle et al. [11] did further research on the performance of the DT-PDE filtering method and proved that this method cannot fully depict the filtering performance that the tensors tried to reflect. Then, they proposed the trace-based tensor-driven PDE, which is equivalent to directed Gaussian filtering. Although it meets the basic demands of multi-scale analysis, the preserving power is insufficient for the corner points of images.

Recently, on the basis of a further analysis of the tensor-driven methods, Tschumperle [12] elucidated that these methods did not possess curvature-preserving ability and therefore proposed the Tensor-driven Curvature-preserving PDE (TCPDE). In this method, diffusion tensors are projected to each direction from 0 to 180°, forming many vector fields, and then traced to a corresponding integral curve in the vector fields for each pixel and Gaussian convolutions of the pixels that the curve pass are computed. Finally, the method calculates the mean linear convolution of the pixel and obtains better smoothing filtering results. It should be pointed out that the TCPDE is equivalent to the Line Integral Convolutions, without considering structure information of the image. To solve this problem, a new Weighted Curvature Preservation PDE (WCPPDE) is proposed. We construct a weighted function with a weighted average strategy which gives different weighted values to each integral curve dependent on the specific image contents and designs the selected method of key parameters of the weighted function. The experimental results indicate the new method has a higher price-to-performance ratio and it can further preserve the image edge information and curvature structure information as well as filtering.

The rest of the paper is organized as follows. Section 1 provides a procedure of the tensor-driven PDE filtering method. Section 2 discusses our method in detail. Section 3 analyzes and compares our method to relative methods based on aspects of the filtering effect and time consumption. Finally, in Section 4, the conclusions of this study are summarized.

2. Tensor-driven curvature-preserving PDE based image filtering method

The equation of the tensor-driven curvature-preserving PDE is shown in (1). In the equation, I represents the image, tr(.) represents the matrix trace, $H = \begin{pmatrix} I_{xx} & I_{xy} \\ I_{xy} & I_{yy} \end{pmatrix}$ is theHessian matrix, I_{xx} represents the second partial in the x direction, $\alpha_\Theta = (\cos\Theta, \sin\Theta)$ is the unit direction angle, D is the diffusion tensor which reflects the image directional information (the detailed computation is shown in Appendix A), $\sqrt{D}\alpha_\Theta = (\upsilon, v)$ representing the projective vector field of D in the α_Θ direction, ∇ is the image gradient, $J_{\sqrt{D}\alpha_\Theta} = \begin{pmatrix} \upsilon_x & \upsilon_y \\ v_x & v_y \end{pmatrix}$ is the Jacobian matrix, and υ_x represents the partial in x direction. Eq. (1) can be manipulated to result in Eq. (2), in which the first term represents diffusion and the second term is the curvature-preserving function which preserves the image corner information.

$$\frac{\partial I}{\partial t} = \frac{2}{\Pi}\int_{\Theta=0}^{\Pi} \mathrm{tr}((\sqrt{\boldsymbol{D}}\boldsymbol{\alpha}_\Theta)(\sqrt{\boldsymbol{D}}\boldsymbol{\alpha}_\Theta)^T\boldsymbol{H}) + (\nabla I)^T \boldsymbol{J}_{\sqrt{\boldsymbol{D}}\boldsymbol{\alpha}_\Theta} d\Theta \tag{1}$$

$$\frac{\partial I}{\partial t} = \mathrm{tr}(DH) + \frac{2}{\Pi}\int_{\Theta=0}^{\Pi} (\nabla I)^T J_{\sqrt{D}\alpha_\Theta} d\Theta \tag{2}$$

$$\frac{\partial I}{\partial t} = \mathrm{tr}(ww^T H) + (\nabla I)^T J_w w \tag{3}$$

$$\frac{\partial I}{\partial t} = \left(\int_{\Theta=0}^{\Pi} W(\sqrt{D}\alpha_\Theta)\cdot(\mathrm{tr}((\sqrt{D}\alpha_\Theta)(\sqrt{D}\alpha_\Theta)^T H)d\Theta + (\nabla I)^T J_{\sqrt{D}\alpha_\Theta}\sqrt{D}\alpha_\Theta)d\Theta\right) \Big/ \int_{\Theta=0}^{\Pi} W d\Theta \tag{4}$$

$$\frac{\partial I}{\partial t} = \frac{\int_{\Theta=0}^{\Pi} W(\sqrt{D}\alpha_\Theta)\cdot(\mathrm{tr}((\sqrt{D}\alpha_\Theta)(\sqrt{D}\alpha_\Theta)^T H)d\Theta)}{\int_{\Theta=0}^{\Pi} W(\sqrt{D}\alpha_\Theta)d\Theta} + \frac{\int_{\Theta=0}^{\Pi} W(\sqrt{D}\alpha_\Theta)\cdot(\nabla I)^T J_{\sqrt{D}\alpha_\Theta}\sqrt{D}\alpha_\Theta d\Theta}{\int_{\Theta=0}^{\Pi} W(\sqrt{D}\alpha_\Theta)d\Theta}. \tag{5}$$

Based upon further analysis of Eq. (1), the diffusion tensor D is projected in each direction of $[0, \Pi)$. W denotes the projective vector field as D projects and the corresponding diffusion equation (3) is obtained. Due to the vector field w, which is obtained from the diffusion vector, D projects in each direction and will indefinitely point at an image edge or gradient. In other words, the diffusion equation of each vector field w affects the filtering result differently. Compared to Eqs. (1) and (2), the TCPDE averages diffusion equations for every direction, which influences the ability to preserve the