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An improved two-point velocity method for estimating the roughness coefficient of natural channels

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ABSTRACT

Previous studies for determining the roughness coefficient are applicable to problems similar to the specific physical conditions under which the guidance is developed. The present study attempts to find a general relationship between Manning's *n* and velocity distribution and determines the roughness coefficient based on the theoretical velocity distribution of natural channels. In this study, an equation is developed for estimating Manning's roughness coefficient using cross-sectional data and velocity observations without slope observations. The simple measurement of stream flow is usually made by taking velocity measurement at several verticals at two-tenths and eight-tenths of the total channel depth. These measurements, averaged to give the mean velocity in the vertical, can be used to estimate Manning's *n*. It provides an easy way to determine the roughness of streams using cross-sectional data and velocity observations, and therefore eliminating the need for slope observations. It is concluded that this equation for computing Manning's *n* in natural channels gives stable results and satisfactory accuracy.

1. Introduction

In applying the Manning formula, the greatest difficulty lies in the estimation of Manning's roughness coefficient *n* as there is no exact method for determining it. At the present stage of knowledge, four general approaches are taken for selecting the *n* value: (1) consulting tables or photographs of channel reaches of typical *n* values which can be used to estimate *n* for a different reach with recognizably similar characteristics (Chow, 1959; Barnes, 1967; Limetinos, 1970); (2) measuring friction slopes, discharges and some cross-sections which is both time consuming and expensive; (3) adopting empirical formulas to estimate the values of roughness based on the particle size distribution curve of surface bed material (French, 1985; Henderson, 1966); or (4) adopting empirical formulas to estimate the values of roughness based on friction slope or water surface slope (Riggs, 1976; Bray, 1979; Dingman and Sharma, 1997). However each of these contributions is only applicable to problems similar to the specific physical conditions under which the guidance is developed and even then their accuracy is still questionable.

Although discharges are measured at one cross-section when the hydraulic method is applied, one still cannot directly calculate the roughness value if the slope is unknown. When the water surface slope, the energy slope and the slope of the channel bottom do not parallel, the computation of roughness by the Manning formula is not appropriate even after the water surface slope is obtained. However, a simple method to measure stream flow is to measure velocity at several verticals at two-tenths and eight-tenths of the total depth. These measurements, averaged to give the mean velocity of the vertical, can be used to estimate Manning's *n* for a wide rough channel utilizing logarithmic velocity distribution. Chow (1959), and French (1985) applied this method to wide rough channels. Nguyen and Fenton (2004) applied this method to three rivers in Victoria and presented a sensitivity analysis. In this paper, the two-point velocity method is verified and extended to four natural mountain streams using mean depth and averaged velocity to estimate the value of the roughness coefficient.

2. Velocity distribution in turbulent flow

In a uniform flow the tractive force is apparently equal to the effective component of the gravity force acting on the body of water, parallel to the channel bottom and equal to *wALS*, where *w* is the unit weight of water, *A* is the wetted area, *L* is the length of the channel reach, and *S* is the slope. Thus, the average value of the tractive force per unit wetted area, or the so-called unit tractive force τ_0, is equal to $wALS/PL = wRS$, where *P* is the wetted perimeter and *R* is the hydraulic radius that is

$$\tau_0 = wRS \tag{1}$$

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Nomenclature

n	Manning's roughness coefficient
V	cross-sectional average velocity of the flow
R	hydraulic radius
S	energy slope
g	gravitational acceleration
w	unit weight of water
A	wetted area
L	length of the channel reach
τ_0	unit tractive force
P	wetted perimeter
ρ	mass density
l	characteristic length known as the mixing length
dv/dy	velocity gradient at a normal distance y from the solid surface
k	constant of proportionality between l and y
y_0	constant of integration
V_f	friction velocity or shear velocity
h	depth of water
B	length of the curve of equal velocity
y	vertical depth measured from the boundary to the curve of equal velocity
γ	function depending on the shape of the section
$v_{0.2}$	velocity at two-tenths the depth
$v_{0.8}$	velocity at eight-tenths the depth
k_u	$v_{0.2}/v_{0.8}$
C	Chezy's C

In a wide open channel, the hydraulic radius is equal to the depth of flow y; hence $\tau_0 = wyS$.

The velocity distribution in a uniform channel flow becomes stable after the turbulent boundary layer has fully developed. In the turbulent boundary layer, the distribution can be shown to be approximately logarithmic.

The shearing stress at any point in a turbulent flow moving over a solid surface has been given by Prandtl as

$$\tau = \rho l^2 \left(\frac{dv}{dy}\right)^2 \tag{2}$$

In which ρ, mass density, w/g, where w is the unit weight of the fluid and g is the gravitational acceleration; l, a characteristic length known as the mixing length; and dv/dy, velocity gradient at a normal distance y from the solid surface.

For the region near the solid surface, Prandtl introduced two assumptions: (1) that the mixing length is proportional to y, and (2) that the shearing stress is constant. Since the shearing stress at the channel surface is equal to the unit tractive force, the second assumption gives $\tau = \tau_0$. From these two assumptions, Eq. (2) may be written

$$dv = \frac{1}{k}\sqrt{\frac{\tau_0}{\rho}}\frac{dy}{y} \tag{3}$$

in which k is the von Karman coefficient. The value of k here is determined by Nikuradse's experiments to be about 0.40. Integrating Eq. (3),

$$v = 5.75\sqrt{\frac{\tau_0}{\rho}}\log\frac{y}{y_0} \tag{4}$$

in which y_0 is a constant of integration.

From Eq. (1) and $w = \rho g$, it can be shown that

$$\sqrt{\frac{\tau_0}{\rho}} = \sqrt{gRS} = V_f \tag{5}$$

The quantity represented by V_f has the dimension of a velocity. Since it varies with the boundary friction τ_0, it is known as the friction velocity or shear velocity. Thus, Eq. (4) may be written

$$v = 2.5V_f \ln\frac{y}{y_0} \tag{6}$$

This equation indicates that the velocity in the turbulent region is a logarithmic function of the distance y. It is commonly known as the Prandtl–von Karman universal-velocity-distribution law.

3. Theoretical uniform equations

By the continuity equation, the total discharge through an ordinary channel section (Fig. 1) may be written

$$Q = AV = \int_{\delta_0=0}^{y=h} v\,dA = \int_0^h vB\,dy \tag{7}$$

where h is the depth of water, A is the water area, B is the length of the curve of equal velocity, and y is the vertical depth measured from the boundary to the curve of equal velocity. Since the laminar sublayer is relatively very thin, δ_0 can be assumed to be zero. It is further assumed that the maximum velocity is at the free surface and that the length B is proportional to its vertical distance y from the boundary that is

$$B = P - \gamma y \tag{8}$$

where P is the wetted perimeter and γ is a function depending on the shape of the section. Thus, the water area is equal to

$$A = \int_0^h B\,dy = Ph - \frac{\gamma}{2}h^2 \tag{9}$$

Substituting in Eq. (7) the value of v from Eq. (6), B from Eq. (8), and A from Eq. (9) and then integrating and simplifying, the following equation is obtained:

$$V = V_f\left\{5.75\log\left[\frac{h}{mR}\exp\left(-1-\frac{\gamma h^2}{4A}\right)\right] + 5.75\log\frac{mR}{y_0}\right\} \tag{10}$$

In the above equation the quantity represented by the first term on the right-hand side is a function of the shape of the channel section. However, the variation of this quantity with different shapes of the section is relatively small. For the sake of simplicity, the quantity may be represented by an over-all constant A_0. This constant includes not only the shape function but also other uncertain factors such as the effect of free surface and the effect of nonuniform distribution of the tractive force at the boundary. Accordingly, Eq. (10) may be written

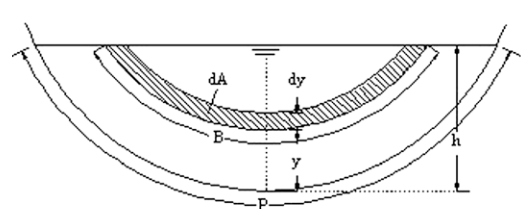

Fig. 1. Channel section to illustrate notation.