Computers and Mathematics with Applications 64 (2012) 1789–1803

Contents lists available at SciVerse ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

# Differential subordination and superordination results for Cho–Kwon–Srivastava operator

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ARTICLE INFO

*Article history:*
Received 11 October 2011
Received in revised form 24 January 2012
Accepted 2 February 2012

*Keywords:*
Analytic
Univalent
Subordination
Superordination

ABSTRACT

Making use of the concepts of differential subordination and superordination, many subordination and superordination results are obtained for analytic functions in the open unit disk using the Cho–Kwon–Srivastava operator by investigating appropriate classes of admissible functions. Sandwich-type results are also obtained.


## 1. Introduction

Let $\mathscr{H}(\mathbb{U})$ be the class of functions analytic in

$$\mathbb{U} := \{z \in \mathbb{C} : |z| < 1\}$$

and $\mathscr{H}[a, n]$ be the subclass of $\mathscr{H}(\mathbb{U})$ consisting of functions of the form

$$f(z) = a + a_n z^n + a_{n+1} z^{n+1} + \cdots,$$

with $a \in \mathbb{C}$, $\mathscr{H}_0 \equiv \mathscr{H}[0, 1]$ and $\mathscr{H} \equiv \mathscr{H}[1, 1]$. Let $\mathscr{A}_p$ denote the class of all analytic functions of the form

$$f(z) = z^p + \sum_{k=p+1}^{\infty} a_k z^k \quad (z \in \mathbb{U}) \tag{1}$$

and let $\mathscr{A}_1 := \mathscr{A}$.

Let $f$ and $F$ be members of $\mathscr{H}(\mathbb{U})$. The function $f(z)$ is said to be subordinate to $F(z)$, or $F(z)$ is said to be superordinate to $f(z)$, if there exists a function $w(z)$ analytic in $\mathbb{U}$ with

$$w(0) = 0 \quad \text{and} \quad |w(z)| < 1, \quad (z \in \mathbb{U}),$$

such that $f(z) = F(w(z))$. In such a case we write $f(z) \prec F(z)$. If $F$ is univalent, then

$$f(z) \prec F(z) \quad \text{if and only if} \quad f(0) = F(0)$$

and $f(\mathbb{U}) \subset F(\mathbb{U})$.

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0898-1221/$ – see front matter 
doi:10.1016/j.camwa.2012.02.016

For two functions $f(z)$ given by (1) and $g(z) = z^p + \sum_{k=p+1}^{\infty} b_k z^k$, the Hadamard product (or convolution) of $f$ and $g$ is defined by

$$(f * g)(z) := z^p + \sum_{k=p+1}^{\infty} a_k b_k z^k := (g * f)(z). \tag{2}$$

For a function $f \in \mathscr{A}_p$, given by (1), it follows from

$$I_p^{\lambda}(a, c)f(z) = \phi_p^{(+)}(a, c; z) * f(z), \quad z \in \mathbb{U}$$

that for $\lambda > -p$ and $a, c \in \mathbb{R} \setminus \mathbb{Z}_0^-$, where

$$\phi_p(a, c; z) * \phi_p^{+}(a, c; z) = \frac{z^p}{(1-z)^{\lambda+p}}$$

and

$$\phi_p(a, c; z) = z^p + \sum_{k=1}^{\infty} \frac{(a)_k}{(c)_k} z^{p+k}.$$

$$\begin{aligned} I_p^{\lambda}(a, c)f(z) &= z^p + \sum_{k=1}^{\infty} \frac{(c)_k(\lambda+p)_k}{(a)_k(1)_k} a_{p+k} z^{p+k} \\ &= z^p {}_2F_1(c, \lambda+p, a; z) * f(z), \quad z \in \mathbb{U}. \end{aligned} \tag{3}$$

From (3), we deduce that

$$z(I_p^{\lambda}(a, c)f(z))' = (\lambda+p)I_p^{\lambda+1}(a, c)f(z) - \lambda I_p^{\lambda}(a, c)f(z) \tag{4}$$

and

$$z(I_p^{\lambda}(a+1, c)f(z))' = aI_p^{\lambda}(a, c)f(z) - (a-p)I_p^{\lambda}(a+1, c)f(z). \tag{5}$$

We also note that

$$\begin{aligned} &I_p^0(p+1, 1)f(z) = p\int_0^z \frac{f(t)}{t}dt, \\ &I_p^0(p, 1)f(z) = I_p^1(p+1, 1)f(z) = f(z), \\ &I_p^1(p, 1)f(z) = \frac{zf'(z)}{p}, \\ &I_p^2(p, 1)f(z) = \frac{2zf'(z) + z^2 f''(z)}{p(p+1)}, \\ &I_p^2(p+1, 1)f(z) = \frac{f(z) + zf'(z)}{p+1}, \\ &I_p^n(a, a)f(z) = D^{n+p-1}f(z), \quad n \in \mathbb{N},\ n > -p. \end{aligned}$$

The Ruscheweyh derivative $D^{n+p-1}f(z)$ was introduced by Kim et al. [1] and the operator $I_p^{\lambda}(a, c)(\lambda > -p, a, c \in \mathbb{R}\setminus\mathbb{Z}_0^-)$ has been recently introduced by Cho et al. [2]. He investigated (among other things) some inclusion relationships and properties of various subclasses of multivalent functions in $\mathscr{A}_p$, which were defined by means of the operator $I_p^{\lambda}(a, c)$ (see also the related works in [3–9]). For $\lambda = c = 1$ and $a = n + p$, the Cho–Kwon–Srivastava operator $I_p^{\lambda}(a, c)$ yields the Noor integral operator $I_p^1(n+p, 1) = I_{n,p}$ $(n > -p)$ of $(n + p - 1)$ the order, studied by Liu and Noor [10] (see also [11,12]). The linear operator $I_1^{\lambda}(\mu + 2, 1)$ $(\lambda > -1, \mu > -2)$ has also been recently introduced and studied by Choi et al. [13] (see also the works of Srivastava et al. [14–16]). For relevant details about further special cases of the Choi–Saigo–Srivastava operator $I_1^{\lambda}(\mu + 2, 1)$, the interested reader may refer to the works by Cho et al. [2,17], Choi et al. [13], Aouf and Srivastava [18] and Liu and Srivastava [19].

In an earlier investigation, a sequence of results using differential subordination with convolution for the univalent case has been studied by Shanmugam [20]. A systematic study of the subordination and superordination using certain operators under the univalent case has also been studied by Shanmugam et al. [21–25]. We observe that for these results, many of the investigations have not yet been studied by using appropriate classes of admissible functions. The motivation of this work is to obtain various differential subordination and superordination results for analytic functions in the open unit disk using the Cho–Kwon–Srivastava operator by investigating appropriate classes of admissible functions. Sandwich-type results are also obtained.