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Precise asymptotics for the linear processes generated by associated random variables in Hilbert spaces[☆,☆☆]

Ke-Ang Fu[a,*], Jie Li[b], Ya-Juan Dong[a], Hui Zhou[c]

[a] School of Statistics and Mathematics, Zhejiang Gongshang University, Hangzhou 310018, China
[b] School of Mathematics and Statistics, Zhejiang University of Finance and Economics, Hangzhou 310018, China
[c] College of Economics, Hangzhou Dianzi University, Hangzhou 310018, China

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ABSTRACT

Let $\{\varepsilon_k, k \in \mathbf{Z}\}$ be a strictly stationary associated sequence of random variables taking values in a real separable Hilbert space, and $\{a_k; k \in \mathbf{Z}\}$ be a sequence of bounded linear operators. For a linear process $X_k = \sum_{i=-\infty}^{\infty} a_i(\varepsilon_{k-i})$, the precise probability and moment convergence rates of $\sum_{i=1}^{n} X_i$ in some limit theorems are discussed.

1. Introduction and main results

Let $\mathbf{H}$ be a separable real Hilbert space with the norm $\|\cdot\|$ generated by an inner product, $\langle\cdot,\cdot\rangle_H$ and let $\{e_i; i \geq 1\}$ be an orthonormal basis in $\mathbf{H}$. Let $L(\mathbf{H})$ be the class of bounded linear operators from $\mathbf{H}$ to $\mathbf{H}$ and denote by $\|\cdot\|_{L(\mathbf{H})}$ its usual uniform norm. Let $\{\varepsilon_k, k \in \mathbf{Z}\}$ be a sequence of $\mathbf{H}$-valued random variables, and $\{a_k, k \in \mathbf{Z}\}$ be a sequence of operators, $a_k \in L(\mathbf{H})$. Define the stationary Hilbert space process by

$$X_k = \sum_{i=-\infty}^{\infty} a_i(\varepsilon_{k-i}), \quad k \in \mathbf{Z}, \tag{1.1}$$

provided the series is convergent in some sense. The sequence $\{X_k, k \in \mathbf{Z}\}$ is a natural extension of the multivariate linear processes [1]. These types of processes with values in functional spaces also facilitate the study of estimation and forecasting problems for several classes of continuous time processes, and one can refer [2] for more details.

It is noted that when $\{\varepsilon_k, k \in \mathbf{Z}\}$ is a strong $\mathbf{H}$-white noise (i.e. a sequence of i.i.d. $\mathbf{H}$-valued random variables such that $0 < \mathsf{E}\|\varepsilon_k\|^2 < \infty$ and $\mathsf{E}\varepsilon_k = 0$), the series in (1.1) converges almost surely and in $L_1(\mathbf{H})$, and $S_n = \sum_{i=1}^{n} X_i$ satisfies the central limit theorem, provided $\sum_{i=-\infty}^{\infty} \|a_i\|_{L(\mathbf{H})} < \infty$ [3,4]. Moreover, Bosq [5] established a Berry–Esseen type inequality with an additional condition $\sum_{i=1}^{\infty} i\|a_i\|_{L(\mathbf{H})} < \infty$.

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* Corresponding author.
E-mail address: fukeang@hotmail.com (K.-A. Fu).

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Recently, some researchers have investigated the limit theorems of the linear process X_k by assuming that $\{\varepsilon_k, k \in \mathbf{Z}\}$ is a strictly stationary sequence of (negatively) associated $\mathbf{H}$-valued random variables, which extend many previous results. For example, Ko and Kim [6] studied the functional central limit theorem.

Before stating their results, we first introduce the notions of associated random variables, associated random vectors and $\mathbf{H}$-valued associated random variables (See [7,6,8], respectively).

Definition 1.1. A finite sequence of real-valued random variables $\{X_k; 1 \le k \le n\}$ is said to be associated, if

$$\mathrm{Cov}\{f(X_1, \ldots, X_n), g(X_1, \ldots, X_n)\} \ge 0,$$

whenever f and g are coordinatewise increasing and the covariance exists. An infinite sequence of random variables is associated if every finite subsequence is associated.

Definition 1.2. A finite sequence of R^d-valued random vectors $\{X_k; 1 \le k \le n\}$ is said to be associated, if for all coordinatewise increasing functions $f, g : R^{nd} \to R$

$$\mathrm{Cov}\{f(X_1, \ldots, X_n), g(X_1, \ldots, X_n)\} \ge 0,$$

whenever the covariance exists. An infinite sequence of random vectors is associated if every finite subsequence is associated.

Definition 1.3. A sequence of $\mathbf{H}$-valued random variables $\{X_k; k \ge 1\}$ is said to be associated, if for some orthonormal basis $\{e_i; i \ge 1\}$ in $\mathbf{H}$ and for any $d \ge 1$, the d-dimensional sequence $(\langle X_i, e_1\rangle_H, \ldots, \langle X_i, e_d\rangle_H)$, $i \ge 1$, is associated.

The main result of Ko and Kim [6] reads as follows.

Theorem A. *Let X_k be an $\mathbf{H}$-valued linear processes given by (1.1), where $\{a_k, k \in \mathbf{Z}\}$ is a sequence of linear bounded operator satisfying $\sum_{i=-\infty}^{\infty} \|a_i\|_{L(\mathbf{H})} < \infty$, and $\{\varepsilon_k; k \in \mathbf{Z}\}$ is a strictly stationary associated sequence of $\mathbf{H}$-valued random variables with $\mathrm{E}\varepsilon_1 = 0$ and $0 < \mathrm{E}\|\varepsilon_1\|^2 < \infty$. If $\tau^2 := \mathrm{E}\|\varepsilon_1\|^2 + 2\sum_{i=2}^{\infty} \mathrm{E}(\langle \varepsilon_1, \varepsilon_i\rangle_{\mathbf{H}}) < \infty$, then we have*

$$n^{-1/2} \sum_{i=1}^{[nt]} X_i \to W \quad \textit{in distribution},$$

where W is a Wiener process on $\mathbf{H}$ with covariance operator $A\Gamma A^*$, $A = \sum_{i=-\infty}^{\infty} a_i$, A^* is the adjoint operator of A, $\Gamma = (\tau_{k\ell})$, $k, \ell = 1, 2, \ldots$, and

$$\tau_{k\ell} = \mathrm{E}(\langle e_k, \varepsilon_1\rangle_{\mathbf{H}} \langle e_\ell, \varepsilon_1\rangle_{\mathbf{H}}) + \sum_{i=2}^{\infty} [\mathrm{E}(\langle e_k, \varepsilon_1\rangle_{\mathbf{H}} \langle e_\ell, \varepsilon_i\rangle_{\mathbf{H}}) + \mathrm{E}(\langle e_\ell, \varepsilon_1\rangle_{\mathbf{H}} \langle e_k, \varepsilon_i\rangle_{\mathbf{H}})].$$

Inspired by them, in this paper we aim to further study the limit properties of linear processes generated by dependent $\mathbf{H}$-valued random variables, and the exact probability and moment convergence rates of S_n in some limit theorems are derived.

Let $\{\varepsilon_k; k \in \mathbf{Z}\}$ be a strictly stationary sequence of associated $\mathbf{H}$-valued random variables. Let G be an $\mathbf{H}$-valued Gaussian random variable with mean zero and covariance $A\Gamma A^*$. Denote the largest eigenvalue of $A\Gamma A^*$ by σ^2. Let l be the dimension of the corresponding eigenspace, and let σ_i^2, $1 \le i \le l'$ be the positive eigenvalues of $A\Gamma A^*$ arranged in a nonincreasing order and take into account the multiplicities. Further, if $l' < \infty$, put $\sigma_i^2 = 0$, $i \ge l'$. Note that we always have $\sigma_i^2 = \sigma^2$, $1 \le i \le l$ and $\sigma_i^2 < \sigma^2$, $i > l$ [9].

Now it is in a position to state our main results.

Theorem 1.1. *Let X_k be an $\mathbf{H}$-valued linear processes given by (1.1), where $\{a_k, k \in \mathbf{Z}\}$ and $\{\varepsilon_k; k \in \mathbf{Z}\}$ are defined as Theorem A. Then under the assumptions of Theorem A, we have that for any $\delta > -1$,*

$$\lim_{\epsilon \searrow 0} \epsilon^{2(\delta+1)} \sum_{n=1}^{\infty} \frac{(\log\log n)^\delta}{n \log n} \mathrm{P}(\|S_n\| \ge \epsilon\sigma\sqrt{2n\log\log n}) = \frac{\mathrm{E}\|G\|^{2(\delta+1)}}{(\delta+1)(2\sigma^2)^{(\delta+1)}}, \tag{1.2}$$

$$\lim_{\epsilon \searrow 0} \epsilon^{2(\delta+1)} \sum_{n=1}^{\infty} \frac{(n \log n)^\delta}{n} \mathrm{P}(\|S_n\| \ge \epsilon\sigma\sqrt{n\log n}) = \frac{\mathrm{E}\|G\|^{2(\delta+1)}}{(\delta+1)(2\sigma^2)^{\delta+1}},$$

Theorem 1.2. *Under the conditions of Theorem 1.1, we have that for any $\delta > -1/2$*

$$\lim_{\epsilon \searrow 0} \epsilon^{2\delta+1} \sum_{n=1}^{\infty} \frac{(\log\log n)^{\delta-1/2}}{n^{3/2} \log n} \mathrm{E}\{\|S_n\| - \epsilon\sigma\sqrt{2n\log\log n}\}_+ = \frac{\mathrm{E}\|G\|^{2(\delta+1)}}{(\delta+1)(2\delta+1)(2\sigma^2)^{\delta+1/2}}, \tag{1.3}$$

$$\lim_{\epsilon \searrow 0} \epsilon^{2\delta+1} \sum_{n=1}^{\infty} \frac{(\log n)^{\delta-1/2}}{n^{3/2}} \mathrm{E}\{\|S_n\| - \epsilon\sigma\sqrt{n\log n}\}_+ = \frac{\mathrm{E}\|G\|^{2(\delta+1)}}{(\delta+1)(2\delta+1)(2\sigma^2)^{\delta+1/2}},$$