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Dynamic bidding analysis in power market based on the supply function[☆]

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ABSTRACT

This paper presents a dynamic bidding model of the power market based on the Nash equilibrium and a supply function. The new model is composed of different dynamic systems and semismooth equations by means of the nonlinear complementarity method. Comparing with those existing bidding models, the remarkable characteristic of the new model is twofold: First, it adopts a dynamic bid so that the bidding limit point is the Nash equilibrium point of the market; Second, it considers the system requirement and the market property such as involving the transmission constraints in the network, and using a supply function which is suitable for the oligopolistic competitive power market. All of these imply that the new model is very close to the practical power market. The computation of the dynamic model is discussed by using the semismooth theory. A numerical simulation is presented to test the model behaviors in the uncogestion and the congestion cases, respectively. The numerical tests include the computing behavior of the dynamic model to reach Nash equilibrium points, the influence of the adjusted parameters and the system parameters to the Nash equilibrium, the local stability of the model, and the comparison of simulation effect between the proposed model and the Cournot model. The simulations show that the new bidding model is valid.

1. Introduction

A competitive electricity market includes different participants such as electric power producers, electric consumers and transmission network companies, etc. Under the supply-demand relationship and the unisonal operation of electrical price, generators adjust their own generation quantity, and users change their consumed quantity constantly. The objective of the market is to realize an optimal allocation of the power system resources so that each of the participants can obtain the maximal profit. The dynamic evolvement of an electricity market with the behavior of generation, consumed quantity and power price, includes various messages of the market and operates as the following processes:

- Each participant submits his/her bid to the Independent System Operations (ISO) at a period time t;
- The ISO, taking account of the security of transmission network, solves a social cost minimization (or a social welfare maximization) problem to get a dispatch and to announce participants for their production/consumption and prices;
- Each participant operates according to the dispatch of the ISO, and submits the bid of the next period time $t+1$.

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According to the operation mechanism of the power market and the Nash equilibrium concept in economics, the maximal profit problems of the market participants consist of a set of correlative bi-level optimizations (see [1] and references therein). We call the solution set of bi-level optimization problems *Nash Equilibrium Points*. [1] studied the Nash equilibrium problem in the power market, including the existence of solutions and solution approaches. We know that in an actual competitive market environment, the ISO can not completely control the behavior of the market participants, and none of the participants has global information of the market. Both of these imply that in a competitive market it is impossible to obtain Nash equilibrium solutions by solving the Nash equilibrium problem unitedly or independently. Actual methods should be an adjusting process step-by-step with bounded rationality of the market participates so that the market reaches a Nash equilibrium point. This is called a game problem in economics. Our problem in this paper is to set a dynamic bidding manner and to model the power market.

The bidding manner has been extensively studied and various bidding models are presented in the power market (see [2–10] and reference therein). Among the various methods, the simplest way is to estimate the market clearing price of the next time and then present the bid with a lower price than the estimated one. This method is based on an assumption that the market clearing price is not affected by any bidding of the marketers, which is not suitable for the power market since it is controlled by a few big electrical companies and is an oligopolistic competition market. The second method is to estimate the behaviors of the rivals and to present the bid, including conjectured variations [7] and conjectured supply function [11]. The third method is based on the game theory [12] with oligopolistic strategy such as Bertrand model [6], the Cournot model [9], and supply function models [3].

Except for the study of the bidding manners, another key question is the stable property of the power market due to the dynamic action of the marketers. Alvarado et al. [13,14] first studied these issues. By establishing the first-order differential equations with the variables of generation quantities and consumption ones, the stable condition of the power market (also called the dynamic behavior approach to the market equilibrium) is analyzed. [15] also studied the stability of the power market by the controlled method.

We note that most of the strategy bidding methods in the power market rarely considered the transmission constraints since the constraints in the ISO optimization problems will increase the analysis difficulty. In order to set a strategy bid close to the practical systems and the market, based on the Cournot model, [9] proposed a dynamic bidding model by combining the transmission constraints, and the stability of the model is also studied. We know that the Cournot model is set on the basis of quantity. However, the power market is a competitive market of quantity-price; what's more, it does not suit the case of inelastic demand (or small inelasticity). These motivate us to study a new strategy bidding model of the power market.

Our objective in this paper is to design a new dynamic bidding model according to the characteristics of the power market and to make some analysis on the model. At the end, based on some theories and approaches of the optimization problems, we construct a different dynamic system with constraints of semismooth equations by using the supply function and the nonlinear complementarity problem (NCP) method. The specialties of the proposed dynamic model is twofold. First, it uses the Nash equilibrium theory as a basis. In other words, the final objective of the dynamic bidding is to arrive at the Nash equilibrium point with a suitable dynamic adjustment. Second, the proposed strategy bidding is considered close to the practical power market, including the transmission constraints, being true of the competitive behavior of markets by a supply function which can reflect the potential market power of participants. The model approach is also studied based on the semismooth theory. In order to test the validity of the proposed bidding model, a power network system with three buses/nodes is presented. Numerical simulations analyze the Nash equilibrium points for the two cases of transmission constraints (non-congestion and congestion). Furthermore, the infection of adjusted dynamic parameters and system parameters to Nash equilibrium points is also investigated. Some comparison results with the Cournot model are also presented in simulations. The local stability of the model is studied.

The paper is organized as follows. Section 2 presents the Nash equilibrium mathematical model of the power market. Section 3 constructs the dynamic bidding model based on the Nash equilibrium theory and the optimization theory. Section 4 discusses the approaches to the problems of the new model. In Section 5, numerical simulations for a power network is presented to test the new model. Some final remarks are made in the last Section.

Some notations are used in this paper. $q \equiv (q_0, q_1, \dots, q_N)$ is the generation/consumption quantity at the nodes. Constant vector C is the line limits in the network. We use the index i to express the i-th node, where $a = (a_0, a_1, \dots, a_N)^{\mathrm{T}} \equiv (a_i, a_{-i})^{\mathrm{T}}$ is the bidding vector with $a_{-i} = (a_0, \dots, a_{i-1}, a_{i+1}, \dots, a_N)^{\mathrm{T}}$. α_i is the adjusted parameter in the model of dynamic bidding. $\frac{\partial q_i}{\partial a_i}$ is the generalized derivative in the sense of Clarke [16]. a_{li} and a_{ui} are the bidding bounds. π_i means the profit function. p_i denotes the marginal price. F_i is called the dynamic function used in the dynamic model. The symbol $*$ expresses the corresponding value at the equilibrium point.

2. Nash equilibrium in the power market

This section is a basis on which we construct a dynamic bidding model. Consider an electric network of $N + 1$ nodes, where there is one generator ($i = 0, 1, \dots, s$) and one consumer ($i = s + 1, \dots, N$) at each node. A special node indexed $i = 0$ refers to the reference node (swing bus). The network has a set $\mathcal{L}$ of links, denoted by ij, which connect nodes i and j and have the distribution factor matrix $\Phi = (\phi_{ij})$ on each link ij. Suppose that the power flows are approximated by the DC model.