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Faedo–Galerkin approximate solutions for stochastic semilinear integrodifferential equations☆

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ABSTRACT

In this paper, we consider a class of stochastic semilinear integrodifferential equations and prove the existence, uniqueness and approximate solutions in a separable Hilbert space. The convergence of solutions using Faedo–Galerkin approximations is established. For illustration, an example is worked out.

1. Introduction

In many fields of science and engineering there is a large number of problems which are intrinsically nonlinear and complex in nature, involving stochastic excitations of a Gaussian white noise type. Gaussian white noise is an abstraction and not a physical process, mathematically described as a formal derivative of a Brownian motion process, all such problems are mathematically modelled by stochastic differential equations or in more complicated cases, by stochastic integrodifferential equations. Stochastic differential equations in both finite and infinite dimensions have received much attention in recent years and the existence results may be found in several monograph and books (the reader may refer [1,2] for the finite-dimensional setting and [3,4] for the infinite-dimensional setting). Stochastic integrodifferential equations are important from the viewpoint of applications since they incorporate (natural) randomness into the mathematical description of the phenomena, and therefore provide a more accurate description of it. Since these equations are not solvable in most cases, it is important to find their approximate solutions in an explicit form or in a form suitable for applications of numerical methods. The theory of procedure for approximating the solution of a stochastic differential equations (SDEs) is widely studied by several authors. In the finite-dimensional case, the Cauchy–Maruyama approximation (see Maruyama [5] and Mashane [6]), Caratheodory approximation (see Bell and Mohammed [7] and Mao [8]) and Euler–Maruyama approximation (see [9]) have been used to study the solutions of SDEs. In the infinite-dimensional case, Liu [10] treated the Caratheodory approximation of solutions for a class of semilinear stochastic evolution equation. Barbu [11] considered the solutions of stochastic semilinear equations using Picard approximation. Twardowska [12] gave a wide theory of SDEs concerned with the Wong–Zakai approximation. Langevin et al. [13] investigated, stochastic perturbation of functional differential equations in a Banach space. A semigroup-theoretic development of a theory for the stochastic analogues of deterministic evolution equations is both powerful and beneficial since it enables one to investigate a broad class of stochastic partial differential

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equations within a unified context. In these directions, Caratheodory successive approximate solution has been employed by Balasubramaniam and Dauer [14] for obtaining suitable controllability conditions of semilinear stochastic evolution equations with time delays in Hilbert Space. The results presented in the current manuscript constitute a continuation and generalization of existence results in two ways. For one, we study the Faedo–Galerkin approximations for a class of semilinear stochastic integrodifferential equations, to the authors knowledge this approach is new in the study of such stochastic problems in the literature. And two, our result constitute a stochastic variant of the results concerning the existence of mild solutions in [15]; this enables one to introduce noise into the concrete models that are subsumed as special cases of the integrodifferential system being studied, thereby allowing for a more accurate description of the phenomenon.

In the present work, we shall prove the convergence of solutions of the following stochastic evolution integrodifferential equation in a separable Hilbert space H using the Faedo–Galerkin approximations,

$$\mathrm{d}x(t) + Ax(t)\mathrm{d}t = f(t, x(t))\mathrm{d}t + \int_0^t a(t-s)g(s, x(s))\mathrm{d}w(s), \quad t \in J = (0, T],\ T < \infty$$
$$x(0) = \phi, \tag{1}$$

where A is a closed, positive definite, self-adjoint linear operator from the domain $D(A)$ in H. We assume that A has a pure point spectrum $0 < \lambda_0 \le \lambda_1 \le \cdots$ and a corresponding complete orthonormal system $\{x_i\}$ so that $Ax_i = \lambda_i x_i$ and $(x_i, x_j) = \delta_{ij}$, $(\cdot, \cdot)$ is the inner product in H and δ_{ij} is the Kronecker delta function. These assumptions on A guarantee that A generates an analytic semigroup e^{-tA}. The nonlinear operators f and g are defined on $D(A^\alpha)$ for some α, $0 < \alpha < 1$ and ϕ is in $D(A)$. The map a is a real-valued continuous function on R_+. Suppose $\{w(t)\}_{t\ge 0}$ is a given K-valued Brownian motion or Wiener process with a finite trace nuclear covariance operator $Q \ge 0$. We are employing the same notation $\|\cdot\|$ for the norm $L(K, H)$, where $L(K, H)$ denotes the space of all bounded linear operators from K into H. The functions $f : J \times C_\alpha(T) \to H$ and $g : J \times C_\alpha(T) \to L_Q(K, H)$ are the measurable mappings in H norm $L_Q(K, H)$ norm respectively. ($L_Q(K, H)$ denotes the space of all Q-Hilbert–Schmidt operators from K into H which is going to be defined below).

The manuscript is organized as follows. In Section 2, we recall some necessary preliminaries. In Section 3, using an associated integral equation and projection operator an approximate integral equation is considered. The existence and uniqueness of solution to this approximate integral equations and the convergence of approximate integral equation to the associated integral equation is established. We consider the Faedo–Galerkin approximate solution and prove the main result concerning the convergence of such an approximation in Section 4. Finally in Section 5, an example is presented which illustrates the main theorem.

2. Preliminaries

Let $(\Omega, \mathfrak{F}, P)$ be a complete probability space furnished with complete family of right continuous increasing sub-σ-algebras $\{\mathfrak{F}_t, t \in J\}$ satisfying $\mathfrak{F}_t \subset \mathfrak{F}$. An *H-valued random variable* is an $\mathfrak{F}$-measurable function $x(t) : \Omega \to H$ and a collection of random variables $S = \{x(t, w) : \Omega \to H | t \in J\}$ is called a *stochastic process*. Usually we suppress the dependence on $w \in \Omega$ and write $x(t)$ instead of $x(t, w)$ and $x(t) : J \to H$ in the place of S. Let $\beta_n(t)$ $(n = 1, 2, \ldots)$ be a sequence of real-valued one-dimensional standard Brownian motions mutually independent over $(\Omega, \mathfrak{F}, P)$. For more details of this section the reader may refer [3]. Set

$$w(t) = \sum_{n=1}^{\infty} \sqrt{\lambda_n}\beta_n(t)\zeta_n, \quad t \ge 0,$$

where $\lambda_n \ge 0$, $(n = 1, 2, \ldots)$ are nonnegative real numbers and $\{\zeta_n\}$ $(n = 1, 2, \ldots)$ is complete orthonormal basis in K. Let $Q \in L(K, K)$ be an operator defined by $Q\zeta_n = \lambda_n\zeta_n$ with finite $\mathrm{Tr}\, Q = \sum_{n=1}^{\infty} \lambda_n < \infty$, (Tr denotes the trace of the operator). Then the above K-valued stochastic process $w(t)$ is called a Q-Wiener process. We assume that $\mathfrak{F}_t = \sigma(w(s) : 0 \le s \le t)$ is the σ-algebra generated by w and $\mathfrak{F}_T = \mathfrak{F}$. Let $\varphi \in L(K, H)$ and define

$$\|\varphi\|_Q^2 = \mathrm{Tr}(\varphi Q \varphi^*) = \sum_{n=1}^{\infty} \|\sqrt{\lambda_n}\varphi\zeta_n\|^2.$$

If $\|\varphi\|_Q < \infty$, then φ is called a Q-Hilbert–Schmidt operator. Let $L_Q(K, H)$ denote the space of all Q-Hilbert–Schmidt operators $\varphi : K \to H$. The completion $L_Q(K, H)$ of $L(K, H)$ with respect to the topology induced by the norm $\|\cdot\|_Q$ where $\|\varphi\|_Q^2 = \langle\langle\varphi, \varphi\rangle\rangle$ is a Hilbert space with the above norm topology. The collection of all strongly-measurable, square-integrable H-valued random variables, denoted by $L_2(\Omega, \mathfrak{F}, P; H) \equiv L_2(\Omega; H)$, is a Banach Space equipped with norm

$$\|x(\cdot)\|_{L_2} = (E\|x(.; \nu)\|_H^2)^{\frac{1}{2}},$$

where E stands for integration with respect to probability measure P. An important subspace is given by $L_2^0(\Omega, H) = \{f \in L_2(\Omega, H) : f \text{ is } \mathfrak{F}_0 \text{ measurable}\}$. The fractional power operator A^α of A for $0 \le \alpha \le 1$ are well defined from $D(A^\alpha) \subseteq H$ into H (cf. Pazy [16, p. 69–75]). $D(A^\alpha)$ is a Banach space endowed with norm

$$\|x\|_\alpha = \|A^\alpha x\|, \quad x \in D(A^\alpha).$$

In order to establish the result we assume the following hypotheses.

(H1) $a \in L^p_{loc}(0, \infty)$ for some $1 < p < \infty$.