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On the modelling and simulation of the competition for a secession under media influence by active particles methods and functional subsystems decomposition

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ABSTRACT

This paper deals with the development of a mathematical model for complex socio-economical systems, where external actions play a key role. The aim of the paper is to show the emergence of collective behaviors or macroscopic trends from individual based interactions, where agents are identified by functional subsystems. The approach is based on the methods of the mathematical kinetic theory for active particles, which describes the evolution of large systems of interacting entities which are carriers of specific functions: in our specific application socio-political activities subjected to the influence of the media.

1. Introduction

A new mathematical approach to modelling complex social-economical systems has been recently proposed in [1] as a natural development of the mathematical kinetic theory for active particles [2], already applied in various fields of life sciences, e.g. to model multicellular systems [3], and social behaviors of interacting individuals [4,5]. This mathematical theory describes the evolution of the probability distribution over the microscopic state, called activity, of several interacting entities called active particles. The equation which models the evolution is derived by a conservation balance in the elementary volume of the space of the microscopic states, where the inlet and outlet flows are determined by interactions among active particles.

The above mathematical approach has been developed in [1] to model complex socio-economical systems, where individual behaviors and interactions may play a significant role on the evolution of the system. Indeed, living systems have the ability to think, and as a consequence, to react to external actions, without following rules constant over time. A basic reference is the so called *behavioral economics*, where deterministic rules may be stochastically perturbed by individuals behaviors, that not only show random fluctuations, but may be substantially modified by external environments, e.g. depressive or panic situations. Many attempts to explicate these phenomena can be found in [6–14], where different models of opinion dynamics are described. Other references with a more socio-psychological approach can be found in [15, 16]. Moreover, the literature of the last five years, with works as [17–20], has developed opinion dynamics models, focused to understand the role of extremists in the evolution of the system. These ideas were preliminarly presented in [21].

These classes of models are finalized to show the emergence of collective behaviors and macroscopic trends, as a consequence of individual based interactions. In our approach, the entities which interact to determine the emerging equilibria are ***functional subsystems***: according to the mathematical theory developed in [1], the overall system is decomposed into

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Table 2.1
Competition for a secession under influence of media

Global nation				
Region1		Region2		External action
Political parties	Interest groups	Political parties	Interest groups	Media
$u > 0$ – pro	$u < 0$ – against	$u > 0$ – pro	$u < 0$ – against	Action over u

functional subsystems that refer both to the functions expressed by the socio-economical system, which is the system under consideration, and to the observation and representation scale used in the mathematical modelling process. Therefore each sub-system is related to a specific socio-economical function, called ***activity***. The time-evolution of the whole system is modelled by mathematical differential equations that describe the evolution of the probability distribution over the activity variable. This general mathematical framework is specifically modified in this present paper to model the phenomenon of competition for a secession under the action of media. The analysis is focused on the critical analysis of different strategies to obtain, from the competition, the desired output. Suitable simulations contribute to this investigation.

In detail, the contents of the paper are organized in five sections, which follow the above introduction.

Section 2 deals with a description of the complex socio-economical system under consideration: the competition for a secession under the influence of external actions. Moreover, it is shown how it can be decomposed into several interacting functional subsystems, each of them having the ability to express socio-economical functions and purposes.

Section 3 deals with the derivation of a specific mathematical model for the external action exerted by media. The derivation needs to be developed taking into account the specificity of the role of media action. The model is then inserted within the framework of Section 2.

Section 4 deals with the derivation of a specific model of competition for a secession. The derivation needs, as we shall see, a detailed modelling of the interactions between functional subsystems and among them and the outer environment, namely media. This section finally provides the derivation of a mathematical framework suitable to design specific models.

Section 5 develops simulations focused on analysing the predictive ability of the model with special attention to the role of the external action. Specifically, it is shown how these actions, which are supposed to operate on limited time intervals and have a fixed cost, can modify the output of the competition.

Section 6 critically analyzes some specific applications and provides an overview on research perspectives, and is also focused on further developments of the mathematical model in view of optimization problems.

2. Phenomenological description of the system: Competition for a secession under influence of media

This section deals with the phenomenological description of the socio-economical system under consideration, taking into account the fundamental role of external actions. We propose the same mathematical structure dealt with in [1], focusing on the discrete case.

Before approaching the mathematical modelling details, some introduction is needed. The recent socio-economical literature has shown an increasing interest in the development of models regarding complex political phenomena, as seen in [22–24], even if from a theoretical different approach, which avoid the concept of complex social dynamics. Some interesting references can be found in [25–27], where socio-political phenomena like dictatorships, terrorism and strategical interactions among interests groups are modelled and discussed from a quantitative approach, which takes into account dynamical interactions among agents.

Moreover, while the subject of media influence has been deeply treated in the modern socio-political literature, as witnessed by [28–31], the present literature is lacking the discussion of socio-economical phenomena under the influence of media, from a quantitative approach. The aim of this paper is to propose a first attempt to explain such a complex influence using the theory developed in [1].

The system, which we refer to, is a nation which, as visualized in Table 2.1, is decomposed into two or more subsystems identified by regional interest groups, which express, through specific actions taken by either their political parties or their peculiar interest groups, their attitude towards a process of secession by expressing a function u, which takes negative or positive values: when a certain subsystem is expressing a positive value of u, then it is pro-secession, when it is expressing a negative value of u, then it is against it. The absolute value of u measures the intensity of the function expressed. Another element which plays an important role in the system is the so-called external action, played by media: in our system, each interest group or party can be strongly affected by media, which can be viewed as an additional external functional subsystem; this means that media is another very powerful element of the decomposition.

Bearing this decomposition in mind, the overall nation is viewed as a network of interacting functional sub-systems, each of them corresponding to different interest groups and political parties. Their representation is based on the assumption that each of them has the ability to express a specific function, namely their attitude towards secession. Also the role of the media is initially modelled, considering it as a different kind of subsystem, which can exert an external action.

Let us consider a network of n interacting functional subsystems whose function is identified by the variable u, where the value $u = 0$ separates the positive and negative valued ***functions*** expressed by each subsystem. In particular, the socio-