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An efficient algorithm for solving multi-pantograph equation systems

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ABSTRACT

In this paper, we present a numerical approach for solving the system of multi-pantograph equations with mixed conditions. This system is usually difficult to solve analytically. By expanding the approximate solutions by means of the Bessel functions of first kind with unknown coefficients, the proposed approach consists of reducing the problem to a linear algebraic equation system. The unknown coefficients of the Bessel functions of first kind are computed using the matrix operations of derivatives together with the collocation method. An error estimation is given. The reliability and efficiency of the proposed scheme are demonstrated by some numerical examples. All of the numerical computations have been performed on a computer with the aid of a program written in Matlab.

1. Introduction

In recent years, many authors have studied numerical methods such as the variational iteration method [1], θ-methods [2], the Taylor matrix method [3], the reproducing kernel space method [4], the Adomian decomposition method [5] for approximate solutions of the multi-pantograph equation

$$y'(t) = \lambda y(t) + \sum_{j=1}^{J} \mu_j(t) y(q_j t) + g(t).$$

Additionally, the approximate solutions of generalized pantograph equations have been obtained by using the homotopy method [6], the Taylor polynomial approach [7], the variational iteration method [8], the Bessel collocation method [9], the Taylor method [10,11]. Also, Brunner et al. [12] have used the Galerkin methods for solutions of delayed differential equations of Pantograph type.

Recently, Yüzbaşı et al. [9,13–17] have studied the Bessel matrix and collocation methods for numerical solutions of the neutral delay differential equations, the pantograph equations, the Lane–Emden differential equations, Fredholm integro-differential equations and Volrerra integral and Fredholm integro-differential equation systems.

In this study, we will develop the matrix and collocation methods studied in [9,13–16] for the approximate solutions of the system of multi-pantograph equations

$$\sum_{j=1}^{k} \beta_{i,j}(t) y_j^{(1)}(t) = \sum_{j=1}^{k} \gamma_{i,j}(t) y_j(t) + \sum_{r=1}^{R} \sum_{j=1}^{k} \mu_{i,j}^{r}(t) y_j(q_r t) + g_i(t), \quad i = 1, 2, \ldots, k, \ 0 \le a \le t \le b \tag{1}$$

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with the mixed conditions

$$\sum_{n=1}^{k}(\phi_{n,i}y_n(a)+\psi_{n,i}y_n(b))=\lambda_i,\quad n=1,2,\ldots,k, \tag{2}$$

where $y_j(t),\ (i=1,2,\ldots,k)$ are the unknown functions, $\beta_{i,j}(t),\ \gamma_{i,j}(t),\ \mu^r_{i,j}(t)$ and $g_i(t)$ are the functions defined on interval $a\le t\le b$, and also $q_r,\ \phi_{n,i},\ \psi_{n,i}$ and λ_i are appropriate constants.

Our purpose is to obtain the approximate solutions of system (1) expressed in the truncated Bessel series form

$$y_i(t)=\sum_{n=0}^{N}a_{i,n}J_n(t),\quad i=1,2,\ldots,k \tag{3}$$

so that $a_{i,n},\ n=0,1,2,\ldots,N$ are the unknown Bessel coefficients, N is any chosen positive integer such that $N\ge 1$, and $J_n(t),\ n=0,1,2,\ldots,N$ are the Bessel functions of the first kind defined by

$$J_n(t)=\sum_{k=0}^{\left[\!\left[\frac{N-n}{2}\right]\!\right]}\frac{(-1)^k}{k!(k+n)!}\left(\frac{t}{2}\right)^{2k+n},\quad n\in\mathbb{N},\ 0\le t<\infty.$$

This paper is arranged as follows:

We give some properties of the Bessel functions in Section 2. In Section 3, we introduce the fundamental matrix relations to find the matrix forms of each term of the system (1). The method for gaining approximate solutions is described in Section 4. In Section 5, we present an error estimation for the Bessel polynomial solutions. We illustrate some numerical examples to clarify the method in Section 6. Section 7 presents a brief summary of this article.

2. Some properties of the Bessel functions

The Bessel differential equation is the linear second-order ordinary differential equation given by

$$t^2\frac{d^2y}{dt^2}+t\frac{dy}{dt}+(t^2-p^2)y=0$$

so that p is a non-negative real number. This equation is solved using series solutions. The general solution of this equation has the form

$$y=C_1J_p(t)+C_2Y_p(t)$$

where C_1 and C_2 are constants,

$$J_p(t)=\sum_{k=0}^{\left[\!\left[\frac{N-p}{2}\right]\!\right]}\frac{(-1)^k}{k!(k+p)!}\left(\frac{t}{2}\right)^{2k+p}$$

and

$$\begin{aligned}Y_p(t)=\frac{2}{\pi}\Bigg\{&\left(\ln\left(\frac{t}{2}\right)+\gamma\right)J_p(t)-\frac{1}{2}\sum_{n=0}^{p-1}\frac{(p-n-1)!}{n!}\left(\frac{t}{2}\right)^{2n-p}\\&+\frac{1}{2}\sum_{n=0}^{\infty}(-1)^{n+1}\left(\sum_{k=1}^{n}\frac{1}{k}\sum_{k=1}^{n+p}\frac{1}{k}\right)\left[\frac{1}{n!(n+p)!}\left(\frac{t}{2}\right)^{2n+p}\right]\Bigg\}.\end{aligned}$$

Here, $\gamma\cong 0.5772156$ is Euler's constant and $J_p(t)$ and $Y_p(t)$ are called the Bessel functions of the first kind and the Bessel functions of the second kind [18], respectively.

The orthogonality relation [19] over the interval $[0,b]$ with respect to weight function $w(\rho)=\rho$ is given by

$$\int_0^b J_n\left(\frac{\upsilon_{nm}}{b}\rho\right)J_n\left(\frac{\upsilon_{nk}}{b}\rho\right)\rho d\rho=\begin{cases}0, & m=k\\ \dfrac{b^2}{2}[J_{n+1}(\upsilon_{nm})]^2, & m\ne k\end{cases}$$

where $\rho\in[0,b]$ and υ_{nm} is the mth root of the Bessel function $J_n(t)=0$, i.e. $J_n(\upsilon_{nm})=0$.

The orthogonality relation is used in determining the coefficients in an expansion of a function in terms of a series of Bessel functions.